

Peer-reviewed Conference Contribution

# A hardening plasticity formulation for drained behavior of circular footings

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Upon moving the offshore wind energy sector to deeper waters, there is an increased demand towards developing more complex foundation solutions, in particular suction caisson foundations as single or jacket supported on multiple foundations. Broadly, foundations for offshore wind turbines need to be able to withstand a variety of load combinations throughout their lifetime.

This contribution is devoted to a comprehensive review of the performance of circular surface and shallow foundations under combined loading (VHM), and how this can principally be understood in a theoretical framework in the context of plasticity theory [1-2]. Initially, the associated and non-associated plasticity in offshore foundation design is discussed. The plastic potential function for a non-associated plasticity framework, with the aid of two association factors  $\alpha_h$  and  $\alpha_m$  is as follows:

$$g = \left( \frac{H}{V'h_0\alpha_h} \right)^2 + \left( \frac{M}{V'm_0D\alpha_m} \right)^2 - 2\alpha \frac{H}{V'h_0\alpha_h} \frac{M}{V'm_0D\alpha_m} - \beta_{34} \left( \frac{V}{V'} \right)^{2\beta_3} \left( 1 - \frac{V}{V'} \right)^{2\beta_4} = 0 \quad (1)$$

$$\beta_{34} = \left( \frac{(\beta_3 + \beta_4)^{(\beta_3 + \beta_4)}}{\beta_3^{\beta_3} \beta_4^{\beta_4}} \right)^2$$

where  $V'$  indicates the apex of the potential surface;  $a$  is eccentricity parameter;  $h_0$  and  $m_0$  represent the uppermost size of the yield surface along the vertical loading coordinate;  $\beta_3$  and  $\beta_4$  are curvature factors.

Assuming  $\alpha_h = \alpha_m$ , two potential surfaces with  $V' = V_{peak}$  and  $V' > V_{peak}$  ( $V/V_{peak} = 0.5$ ) have similar shapes despite having different intersection points with the  $V$  axis, a clear indication that associated flow is the governing plastic flow mechanism in radial plane (Fig.1).

Contrarily, the plastic potential surface for surface foundations was plotted for two different association factor values ( $\alpha_h \neq \alpha_m$ , i. e.,  $\alpha_h = 2.5$  and  $\alpha_m = 2.1$ ) while all else remains unchanged. The two cases shown in Fig. 2 are for two different values of  $V'$ . Thus, according to Eq. (1) and Fig. 2, the normality condition is no longer relevant in the radial plane. Further, The hardening law for circular surface footings can be described by the following expression [3], in which the post-peak softening behavior is modeled by a factor  $f_p < 1$ :

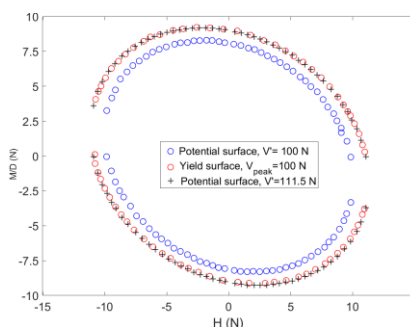


Figure 1: Yield surface and potential function variation as a function of  $V'$  ( $\beta_1 = \beta_1 = 1$ ,  $a = -0.2$ ,  $h_0 = 0.11$ ,  $m_0 = 0.09$ ,  $\alpha_v < 1$ )

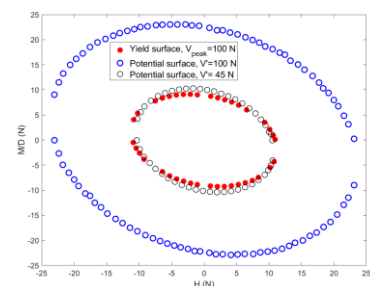
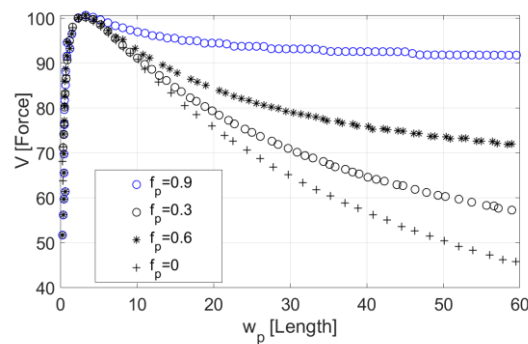


Figure 2: Yield surface and plastic potential functions according to Eq. (1) ( $\alpha_h = 2.5$  and  $\alpha_m = 2.1$ ).

$$V = \frac{k_p w_p + \left( \frac{f_p}{1 - f_p} \right) \left( \frac{w_p}{w_{pm}} \right)^2 V_{peak}}{1 + \left( \frac{k_p w_{pm}}{V_{peak}} - 2 \right) \left( \frac{w_p}{w_{pm}} \right) + \left( \frac{1}{1 - f_p} \right) \left( \frac{w_p}{w_{pm}} \right)^2} \quad (2)$$

The above hardening law describes the relationship between vertical loading and plastic settlement, where  $f_p = V_{post} / V_{peak}$  at the infinity amount of plastic settlement ( $V_{post}$  is the post-peak vertical loading and  $V_{peak}$  is peak bearing capacity), The  $k_p$  denotes initial plastic stiffness,  $w_p$  and  $w_{pm}$  are plastic settlement and plastic settlement at maximum load, respectively (Fig. 3). Although the studies on surface footings yielded  $f_p$  values lower than unity (a clear indication of the prevailing post-peak softening behavior), this key trend is not confirmed in shallow foundations according to FE push-over analyses and scaled model tests [4]. In current study, multiple FE push-over analyses were carried out to establish a new hardening law for a multi-pod system. The validation procedure was detailed in [5-6]. As a consequence,  $f_p = 1.99$  and  $1.85$  and  $k_p = 12700 \text{ kN/m}$  and  $11400 \text{ kN/m}$  corresponding to the embedment ratios 0.5 and 1 were determined, respectively. Furthermore, the initial plastic stiffness was found significantly higher than that of surface footings [7]. Finally, the hardening law and plastic potential surface turned out to be inherently complex, because both involve several parameters that need to be calibrated (i.e., depending strongly on foundation type and soil strength). Thus, special care should be taken when assessing the skirted foundation responses on different loading planes.



**Figure 3: The hardening rule prediction (Eq.2)**

#### Data Availability Statement

All of the data, models, or code generated or used during the study are available from the corresponding author by request.

#### Contributor statement

Amin Barari: Conceptualization, Methodology, Writing - Original Draft.

#### References

- [1] Barari, A., & Ibsen, L.B. (2012). Undrained response of bucket foundations to moment loading. *Applied Ocean Research*, 36, 12-21.
- [2] Ibsen, L.B., Larsen, K.A., & Barari, A. (2014). Calibration of failure criteria for bucket foundations on drained sand under general loading. *Journal of Geotechnical and Geoenvironmental Engineering*, 140(7), 04014033.
- [3] Houlsby G.T., & Cassidy M.J. (2002). A plasticity model for the behaviour of footings on sand under combined loading. *Géotechnique*, 52(2), 117-129.
- [4] Ibsen, L.N., Barari, A., & Larsen, K. (2015). Effect of the embedment on the plastic behavior of bucket foundations. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 141(6), 06015005.
- [5] Barari, A., Glittrup, K., Christiansen, L.R., Ibsen, L.B., & Choo, Y.W. (2021). Tripod suction caisson foundations for offshore wind energy and their monotonic and cyclic responses in silty sand: numerical predictions for centrifuge model tests. *Soil Dynamics and Earthquake Engineering*, 149, 106813.
- [6] Nielsen, K., Svenningsen, J., Barari, A., & Ibsen, L.B. (2023). On the cyclic stability diagram of offshore wind turbine supported on multiple foundations in saturated dense sand. *Engineering Failure Analysis*, 144, 106926.
- [7] Gottardi G., Houlsby G.T. & Butterfield R. (1999). The plastic response of circular footings on sand under general planar loading. *Géotechnique*, 49(4), 453-469.