

Improving Ship Response Estimation with Neural Networks

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ABSTRACT

The feasibility of a data-adaptive multi-fidelity seakeeping model is assessed for use in early stage design in this study. Data adaptive tuning (or correction) of lower-fidelity model predictions are implemented based on training with higher fidelity ship motion response data. Long Short-Term Memory (LSTM) neural networks are incorporated as part of a multi-fidelity approach for prediction of 6 degree of freedom (6-DOF) ship motion responses in waves. LSTM networks are trained and tested with Large Amplitude Motion Program (LAMP) simulations as a target, and SimpleCode simulations and wave time-series as inputs. LSTM networks improve the fidelity of SimpleCode seakeeping predictions relative to LAMP, while retaining the computational efficiency of a lower-fidelity simulation tool.

KEY WORDS

Neural Networks; Seakeeping; Extreme Events.

INTRODUCTION

To understand how a ship will respond in different environmental conditions is vital in early stage design. Consideration of the operating conditions can be made in the form of a developing a database that covers a comprehensive set of ocean wave conditions. Within the database is a collection of ship response statistics as a function of ship heading, speed, and wave conditions. Operational limits are determined based on relationships between these statistics and maximum allowable responses. Generating large databases containing many possible combinations of ship headings, speeds and wave conditions allows for more robust operational guidance ability.

To identify and compare against operational limits, extreme event analysis must be performed. The most straightforward approach to estimating extreme ship response characteristics is through Monte Carlo simulations. However, for most tools of reasonable fidelity, the computational cost is far too expensive when considering potential extreme events for longer return periods and simulation run times on the order of real time. Extrapolation methods, generally based on Weibull distributions, can be explored with a limited dataset. However, this approach requires prior knowledge of the response distribution with particular focus on the tail of the distribution.

Other methods to identify extreme behavior efficiently without overextending assumptions have been developed. One such method is the Design Loads Generator (DLG) (Alford 2008, Kim 2012). DLG was initially developed for linear systems with stochastic Gaussian input, and drew from modified phase distributions based on Extreme Value Theory to generate ensembles of extreme realizations for a given return period.

Another method that has been explored is a lower-fidelity simulation tool that retains major nonlinearities to identify extreme conditions, and then running the identified conditions with a higher-fidelity simulation tool (Reed 2021). In this framework, a surrogate model does not need to be identified but requires a high level of correlation of the peaks between the two simulation tools employed.

An approach with Long Short-Term Memory (LSTM) to correct lower-fidelity ship response data to the level of higher fidelity ship response data was developed in Levine et al. (2024). In the paper, statistics generated by two different training methods were compared between the LSTM and the higher fidelity responses with good results. However, only three degrees-of-freedom (3-DOF) were applied in the ship simulation software solvers.

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In this paper, the method outlined in Levine et al. (2024) is further refined to produce predictions with 6 degrees-of-freedom (6-DOF) simulations. The method applies two seakeeping simulation tools of lower and higher fidelity, which are SimpleCode (Weems and Wundrow 2013) and the Large Amplitude Motion Program or LAMP (Shin et al. 2003), respectively. By running the lower fidelity code (SimpleCode) under the same conditions as the higher fidelity code (LAMP), the motions predicted by SimpleCode can be improved to approximate those from LAMP through a Long Short-Term Memory (LSTM) neural network. However, the forces in the 6-DOF simulations acting on the SimpleCode-simulated ship differ from the LAMP-simulated ship and the simulated ships diverge in space over time. To account for the difference in experienced conditions under the same seaway, a first stage LSTM neural network is introduced to learn the ship path of the LAMP model with the SimpleCode simulation as input. In this approach, all ship motion degrees-of-freedom are considered in the hydrodynamic solvers and are used in the training of the neural networks. After training the LSTM networks, many LAMP-quality ship simulation realizations can be generated with LSTM-corrected SimpleCode results in a much more computationally efficient manner.

In the following sections, the network architecture for training a framework of LSTM networks with SimpleCode as input and LAMP as a target is described. Then, a case study with the David Taylor Model Basin (DTMB) Model 5415 (Moelgaard, 2000) is described and results from the application of the framework are presented.

METHODOLOGY

SimpleCode and LAMP

SimpleCode is a reduced-order seakeeping code that can quickly produce acceptable results (Smith 2019). One of the key simplifications is in the local variation of wave pressure, where the hydrostatic and Froude-Krylov equations can instead use volume integrals rather than integrating over the surface of the ship (Weems and Wundrow 2013). With pre-computed Bonjean curves, the instantaneous submerged volume and geometric center; therefore, sectional hydrostatic and Froude-Krylov forces can be calculated quickly.

LAMP is a higher fidelity code that considers all forces and moments acting on the ship in the time-domain within a 6-DOF, 4th order Runge-Kutta solver (Shin et al. 2003). Central to the code is the solution of the 3-D wave-body interaction problem. Within LAMP, the complexity of this solution can be altered. LAMP-3 is utilized in the current work, where the perturbation velocity potential is solved over the mean wetted hull surface and the hydrostatic and Froude-Krylov forces are solved over the instantaneous wetted hull surface. Additionally, LAMP-3 allows for large lateral motions and forces. LAMP has effectively estimated motions comparable to model tests (Lin and Yue 1991) but is, of course, much more computationally expensive than a lower-fidelity code like SimpleCode. Through some parameters e.g., number of wave frequency components, free surface panel definition, hull offsets, can be altered, LAMP-3 runs at nearly real time i.e., 30 minutes of wall-clock time is required to generate 30 minutes of simulated data. In the same 30 minutes and the same number of frequency components, SimpleCode can produce upwards of 5,000 independent realizations of 30 minutes.

SimpleCode can produce an approximation of LAMP, especially with tuned radiation and diffraction forces included (Weems and Belenky 2018, Pipiras et al. 2022). However, a fidelity gap exists, especially when considering a bimodal wave spectrum. In this study, the 6-DOF implementations of SimpleCode and LAMP were both used. Using the 6-DOF solvers in both SimpleCode and LAMP provides a more accurate representation of the pertinent degrees of freedom as the simulated ship is allowed to move more realistically and the forces act on the ship accordingly. However, only the three vertical degrees of freedom (heave, roll, and pitch) were for comparison. These degrees of freedom have the greatest applicability to the evaluation of typical design criteria.

Long Short-Term Memory (LSTM)

One of the major drivers of the presented method is the Long Short-Term Memory (LSTM) neural network (Hochreiter and Schmidhuber 1997). An LSTM neural network is a recurrent neural network that incorporates both long- and short-term effects that are learned and developed during the training process. These memory effects are stored in weight matrices where they, along with other operations, transform input matrices to the target output matrices. The causal nature of marine dynamics and inclusion of memory effects make LSTM networks particularly well suited for the presented problem. The following set of equations describe the operations that occur in an LSTM layer.

$$f_1 = \sigma(W_{f_1} x^{[t]} + U_{f_1} h^{[t-1]} + b_{f_1}) \quad [1]$$

$$f_2 = \sigma(W_{f_2} x^{[t]} + U_{f_2} h^{[t-1]} + b_{f_2}) \quad [2]$$

$$f_3 = \tanh(W_{f_3} x^{[t]} + U_{f_3} h^{[t-1]} + b_{f_3}) \quad [3]$$

$$f_4 = \sigma(W_{f_4} x^{[t]} + U_{f_4} h^{[t-1]} + b_{f_4}) \quad [4]$$

$$c^{[t]} = f_1 \odot c^{[t-1]} + f_2 \odot f_3 \quad [5]$$

$$h^{[t]} = f_4 \odot \tanh(c^{[t]}) \quad [6]$$

where W and U are weight matrixes, b are the bias vectors, $x^{[t]}$ is the input vector, standardized by the respective standard deviations and means for each input channel, by the respective at time t , $h^{[t]}$ is the hidden state vector at time t , $c^{[t]}$ is the cell state vector at time t , σ is the sigmoid function, $\tanh()$ is the hyperbolic tangent function, and \odot represents the Hadamard product. The output or target at time t is equal to the hidden state vector at time t , $h^{[t]}$. The weight matrices and bias vectors are progressively learned during the training process to minimize the specified loss between the training data and the test data. The present work employs the mean-squared error in Equation 7 to quantify the error between the training and test sets.

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_T(t_i) - y_L(t_i))^2 \quad [7]$$

where N is the number of points in the time series, y is the response matrix which contains the time series of heave, roll, and pitch, subscript T is the target time series, subscript L is the LSTM produced time series, and t_i is the i -th time instant in the time series.

In Levine et al. (2024), the 3-DOF SimpleCode and LAMP simulations were well correlated and the motions and wave elevation time series derived from the SimpleCode simulations could be input into the LSTM with the LAMP heave, roll, and pitch time series as the targets. However, the difference in realized forces acting on the simulated ships in 6-DOF simulations results in a difference in the global position between the SimpleCode and LAMP models. The result is a reduction in correlation between the input and output time series. To account for this lower correlation, a two-step LSTM model is introduced. First, an LSTM framework is trained to transform the 6-DOF SimpleCode motions and wave elevation time series derived from SimpleCode to the surge and sway of the LAMP simulations. These surge and sway time series are combined with the wave spectrum and phases from the given realization to estimate the wave elevation at the center of gravity of the LAMP model. In Levine et al. (2024), an LSTM framework is capable of estimating LAMP motions through a data-driven approach with only the wave elevation as input (“LSTM-Waves”). A similar effort comprises the second step of the LSTM architecture where the wave elevation generated from the estimated LAMP surge and sway are inputs into an LSTM network trained to estimate 6-DOF LAMP heave, roll, and pitch from LAMP-generated wave elevation time series. The framework is illustrated in Figure 1.

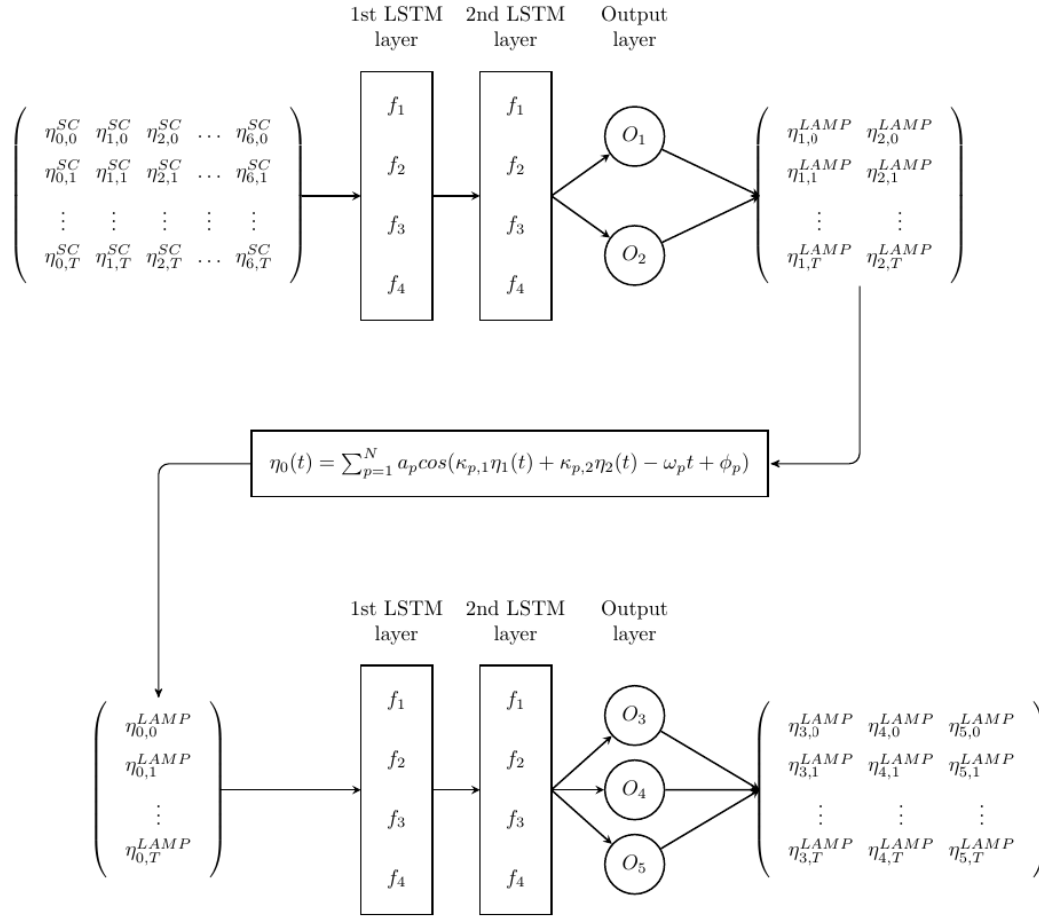


Figure 1: LSTM Two-Step Framework Architecture

Each LSTM architecture consisted of two layers of size 50. The characters and symbols outlined in Figure 1 are described in Table 1. Additionally, superscripts *SC* and *LAMP* indicate that the corresponding time series elements were sourced from SimpleCode or LAMP, respectively. Furthermore, Figure 1 lays out the *training* structure. In testing, the LAMP time series are effectively replaced by LSTM generated time series.

Table 1: LSTM Architecture Parameter Definitions

Parameter Definition	Variable
Input wave at time-step j	$\eta_{o,j}$
Total number time steps	T
i^{th} degree-of-freedom at time-step j	$\eta_{i,j}$
k^{th} gate for LSTM layer	f_k
Number of LSTM units per layer	n
Output layer cell for DOF m	O_m
Number of frequencies in spectrum	N
Wave amplitude for frequency p	a_p
Wave number r for frequency p	$\kappa_{p,r}$
Frequency p	ω_p
Phase for frequency p	ϕ_p

Numerical Experimental Setup

The model hullform for this study was the David Taylor Model Basin (DTMB) Model 5415 (Moelgaard 2000). Figure 2 is a rendering of the DTMB Model 5415 and Table 2 provides the particulars for the vessel.



Figure 2: Rendering of DTMB Model 5415

Table 2: Particulars for DTMB Model 5415

Particular	Symbol	Value
Length between perpendiculars	L_{PP}	142.0 m
Beam	B	19.1 m
Draft	T	6.2 m
Radius of gyration about X-axis	k_{xx}	7.1 m
Radius of gyration about Y-axis	k_{yy}	35.5 m
Vertical center of gravity (w.r.t baseline)	KG	7.5 m
Longitudinal center of gravity (w.r.t midships)	L_{cg}	-0.9 m
Displacement mass	Δ_m	8424.4 t

For this case study, a primary International Towing Tank (ITTC) spectrum (ITTC 2002) characterizing wind-generated waves was applied with $H_s = 4.0$ m and $T_p = 15.0$ s (standard Sea State 5 and most probable modal period) and the relative wave heading set to bow-quartering seas (135°). Additionally, the primary ship speed was set to 10.0 knots. Proportional-integral-derivative (PID) controllers maintained speed and “soft springs” maintained heading in both the SimpleCode and LAMP models.

A total of 50 realizations each 30 minutes in duration were generated in LAMP and SimpleCode. The first stage of the framework to estimate the surge and sway was trained with 30 realizations and validated with 10 realizations. The second stage of the framework to predict the 6-DOF motions was trained with 30 realizations, validated with 10 realizations, and tested with 10 realizations. There was not any testing performed during the first stage as only the output of second stage was compared.

The average standard deviations and time series correlation coefficient for heave, roll, and pitch from the 10 test realizations generated using SimpleCode, LAMP, and the LSTM framework were compared. Equation 8 is the formula for the correlation coefficient.

$$\rho = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} \quad [8]$$

The correlation coefficient is the ratio between the covariance of two random processes, x and y , and the product of the respective standard deviations σ_x and σ_y . To further capture the reliance on strong correlation with LAMP, the relative motion at the starboard bow was also compared. In addition, the absolute relative error was a comparison metric for SimpleCode and the LSTM method to LAMP. The equation for absolute percentage error is as follows:

$$\epsilon = 100\% * \frac{|\hat{X}_L - \hat{X}_E|}{\hat{X}_L} \quad [9]$$

where \hat{X}_L represents the standard deviation of LAMP data and \hat{X}_E represents the standard deviation of the LSTM estimate or SimpleCode.

In addition to the standard deviation statistic, the estimations of peaks between mean up-crossings were compared. The peaks between mean up-crossings from each of the test SimpleCode, LAMP, and LSTM time series were tabulated and probability distribution functions (PDFs) were generated.

RESULTS

Framework Training

To benefit from the neural network framework, it is important to retain computational efficiency along with accuracy compared to the target. Table 3 shows the time necessary to train the network along with time necessary to generate the data. The network training was performed using a NVIDIA Quadro T2000 GPU with 4 GB of memory, the LAMP simulations were performed on a computing cluster using 8 cores each containing 192 GB of memory, and the SimpleCode runs were generated locally on a 32 GB CPU.

Table 3: Computation Time for Data Generation and Network Framework Training

Process	Stage	Computation Time [s]
Data Generation	SimpleCode	37
	LAMP	9,142
Network Training	Surge-Sway Prediction NN	234
	Heave-Roll-Pitch Prediction NN	705

Table 3 shows that the bulk of the process is generating the higher-fidelity LAMP data. After completing the sunk cost of training the framework, much more LAMP-quality data can be generated with the framework and the small cost of producing additional SimpleCode data.

Statistical Comparison

To evaluate the LSTM method relative to LAMP and SimpleCode, the standard deviation from each degree of freedom was estimated from the 10 test realizations. The absolute relative error of the standard deviation compared to LAMP was also calculated to provide a quantitative comparison. Table 4 provides the standard deviation and relative absolute error between LAMP, LSTM, and SimpleCode.

Table 4: Standard Deviation and Absolute Percentage Error to LAMP for Heave, Roll, Pitch, and Starboard Bow Relative Motion

DOF	LAMP	LSTM	LSTM ϵ	SimpleCode	SimpleCode ϵ
Heave [m]	0.472	0.462	2.1%	0.507	7.5%
Roll [deg]	1.397	1.322	5.3%	3.464	147.9%
Pitch [deg]	1.154	1.142	1.0%	1.108	3.9%
Stbd. Bow Relative Motion [m]	1.567	1.547	1.3%	1.887	20.4%

The LSTM method provided an improvement in standard deviation estimate relative to LAMP for each degree-of-freedom. While SimpleCode provides a reasonable estimate in heave and pitch, the roll prediction is significantly over-estimated, which also affects the calculation of the starboard bow relative motion.

While SimpleCode was able to capture generally the heave and pitch statistics, the SimpleCode model diverges spatially in the wave field relative to the LAMP model. As a result, the ship responses between SimpleCode and LAMP are uncorrelated. Table 5 lists the average correlation coefficients for heave, roll, pitch, and the relative motion of the starboard bow between LAMP, LSTM method and SimpleCode.

Table 5: Correlation Coefficients between LAMP, LSTM and SimpleCode for Each Degree of Freedom

DOF	LSTM ρ	SimpleCode ρ
Heave	0.945	0.000
Roll	0.967	0.001
Pitch	0.980	0.001
Stbd. Bow RM	0.964	0.003

The LSTM method shares a very high level of correlation with LAMP across all degrees of freedom. The high level of correlation is imperative in the estimation and identification of extremes or large peak values. In the following section, the peak and time series maxima behavior are investigated.

Peak Behavior

To investigate the behavior of the peaks for LAMP, LSTM, and SimpleCode, the maxima between zero-up-crossings were tabulated across each test realization for each degree of freedom. Figure 3 provides the kernel density estimates of the PDFs for heave, roll, pitch, and relative motion of the starboard bow for LAMP, LSTM, and SimpleCode (SC).

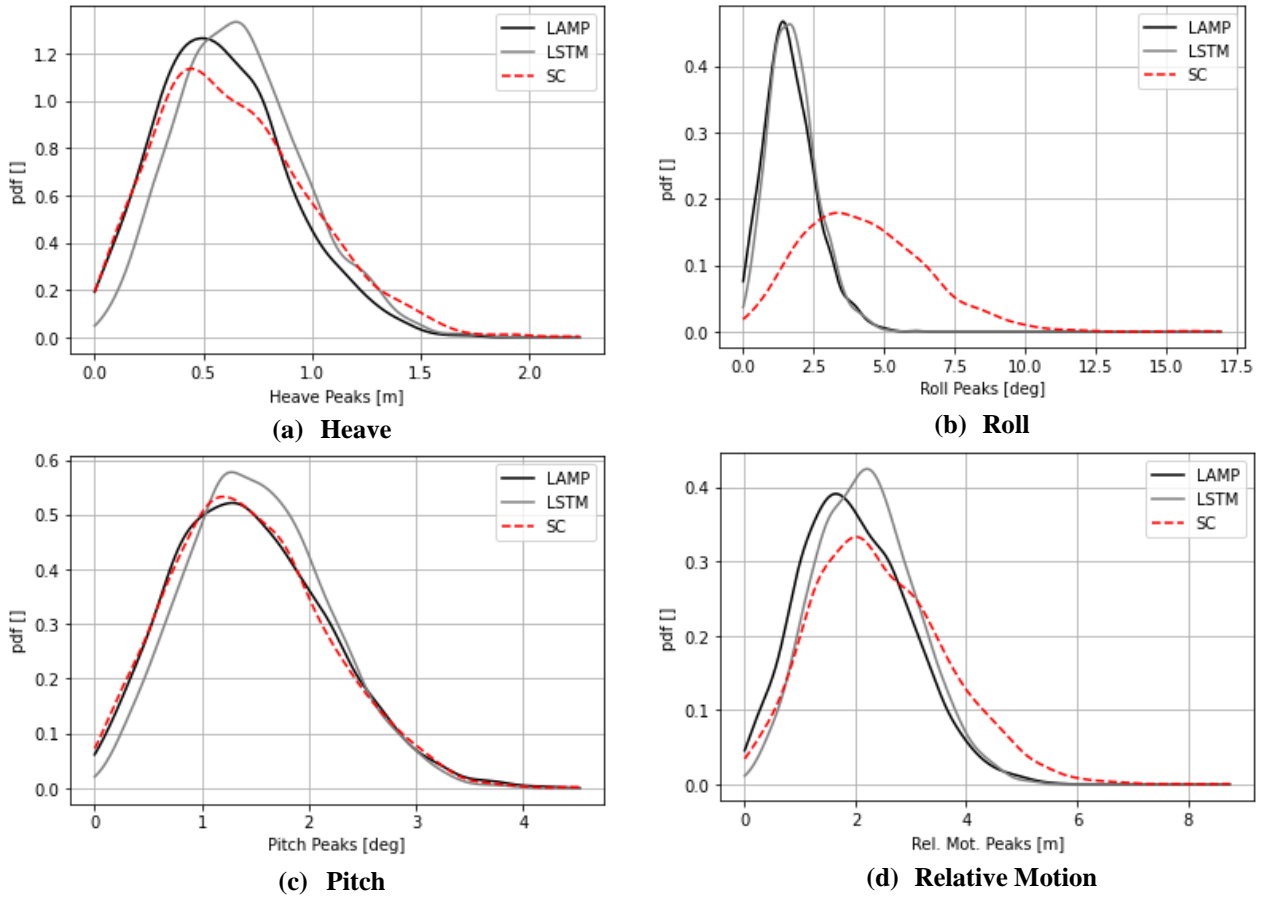


Figure 3: Peak PDF Comparison for LAMP, the LSTM, and SimpleCode

The most probable peak magnitude (MPPM) and 95th percentile values estimated from each PDF in Figure 3 are summarized in Table 6.

Table 6: PDF Characteristic Comparison between LAMP, LSTM, and SimpleCode

DOF	LAMP MPPM	LAMP 95 th	LSTM MPPM	LSTM 95 th	SimpleCode MPPM	SimpleCode 95 th
Heave [m]	0.50	1.14	0.65	1.22	0.44	1.28
Roll [deg]	1.41	3.36	1.65	3.37	3.35	8.22
Pitch [deg]	1.28	2.72	1.28	2.76	1.20	2.73
Stbd. Bow RM [m]	1.65	3.70	2.21	3.77	2.01	4.62

In general, the LSTM method over-estimates the most probable peak magnitude but generally captures the tail behavior produced by the LAMP simulations. The over-estimation of the moderate peaks is likely a result of a high-frequency modulation that was generated in one of the LSTM networks. An example of the modulation is in the pitch time series section in Figure 4.

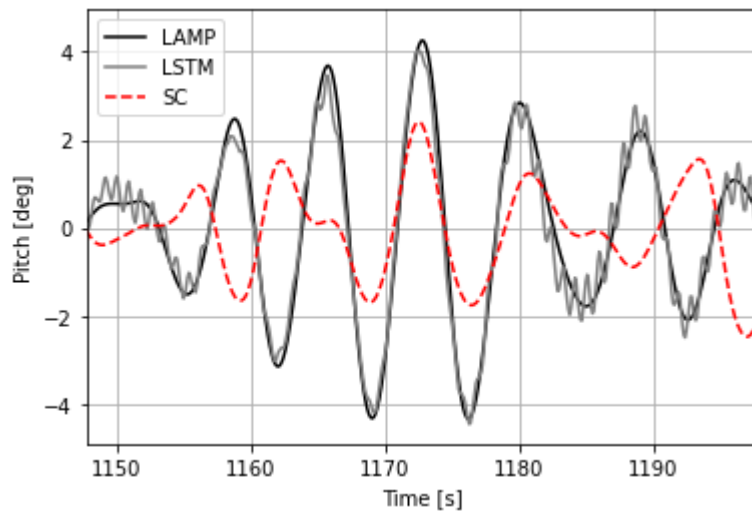
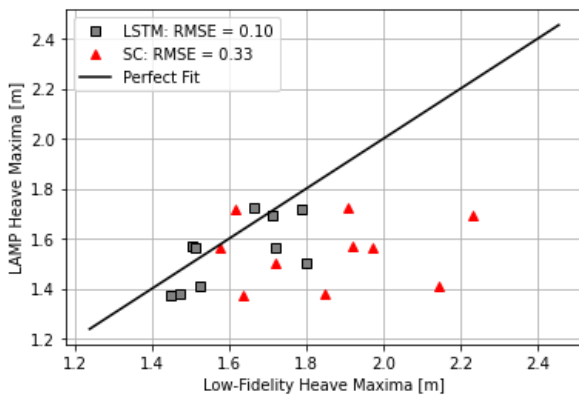
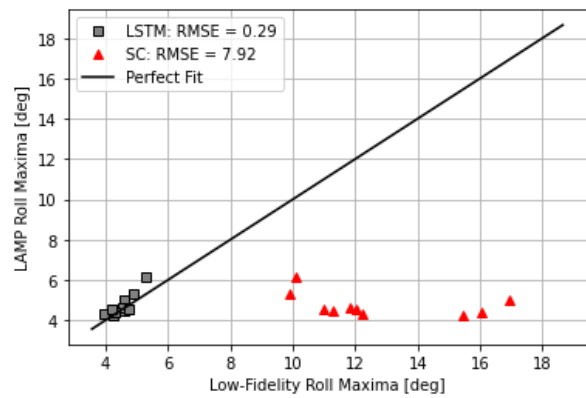


Figure 4: Pitch Time Series of LAMP, the LSTM, and SimpleCode (SC)

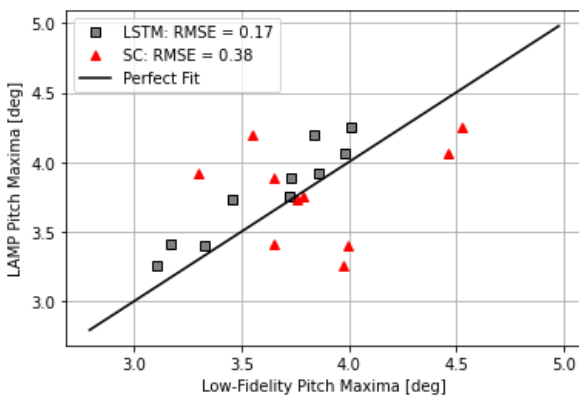
The modulation in Figure 4 near the moderate peaks could likely be attenuated with a low-pass filter. Still, the LSTM method is a good estimator of the peaks, particularly in the tail of the distribution. SimpleCode again is a reliable estimator of heave and pitch. However, the SimpleCode peaks are completely uncorrelated to the LAMP peaks and are not necessarily good predictors for a given realization. To test the predictions across realizations, the time series minima and maxima were gathered from the 10 test realizations for each degree of freedom in SimpleCode and the LSTM and compared directly to the corresponding LAMP time series maxima and minima for the given realization. Figures 5 and 6 show the maxima and minima comparisons for each degree of freedom along with the root mean squared error relative to LAMP.



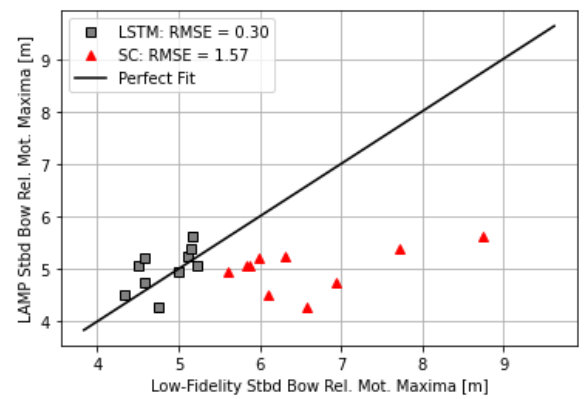
(a) Heave



(b) Roll



(c) Pitch



(d) Relative Motion

Figure 5: Time Series Maxima Comparisons

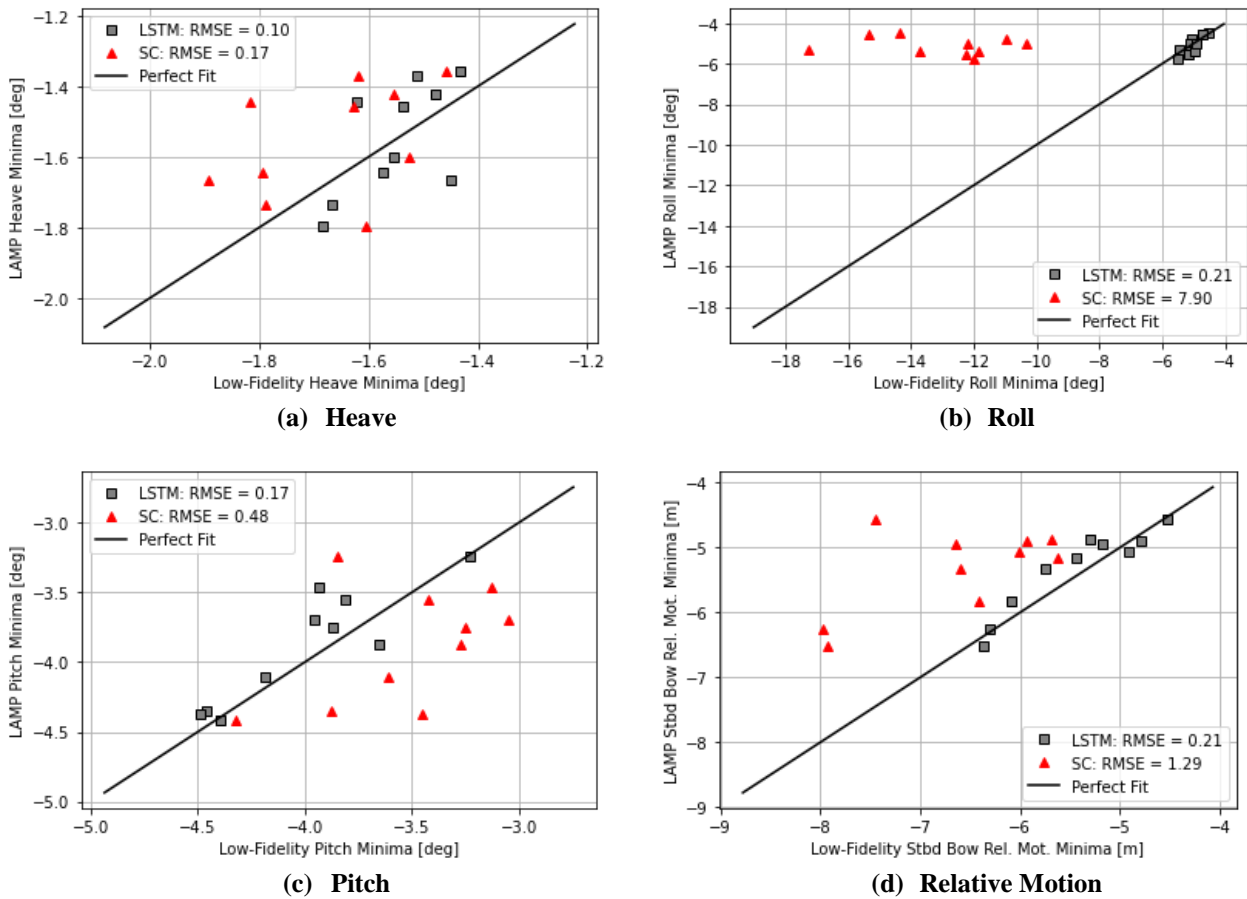


Figure 6: Time Series Minima Comparisons

An important quality of a good qualitative predictor would be for the largest value in the predictor set to line up with the largest value in the test set. The wave realization that produced the largest magnitude LAMP value should also produce the largest LSTM/SimpleCode value. An absolute conclusion cannot necessarily be made with a small dataset of 10 values but the general trend of behaviors can start to be identified. In general, the largest magnitude LSTM events line up with the largest magnitude LAMP events. While the wave field in SimpleCode was produced with the exact same phases, the global position of the SimpleCode model diverges from that of LAMP; therefore, the time series maxima and minima statistics do not necessarily align with that of LAMP. To be able to identify the largest LAMP time series maxima is very important because as the sea state increases, more non-linear effects are included and SimpleCode no longer is a good quantitative estimator of LAMP. Without the ability to estimate the quantitative extremes or identify qualitative extremes, SimpleCode alone cannot reliably identify extreme conditions in early stage design. However, with the LSTM approach, the issue of identifying the qualitative extremes is addressed but further testing must be done in higher sea states for the quantitative extreme estimates.

CONCLUSIONS

An objective of this study was to assess the potential feasibility of a data-adaptive multi-fidelity seakeeping model for use in early stage design. Data adaptive tuning (or correction) of reduced-order model predictions have been implemented based on training with higher fidelity ship motion response data. From these initial results, this approach provides a plausible means for improving the performance of a reduced-order model for ship response estimation.

LSTM neural networks have been incorporated as part of a multi-fidelity approach for prediction of 6-DOF ship motion responses in waves. LSTM networks were trained and tested with LAMP simulations as a target, and SimpleCode simulations and wave time-series as inputs. LSTM networks improve the fidelity of SimpleCode seakeeping predictions relative to LAMP while retaining the computational efficiency of a lower-fidelity simulation tool.

The LSTM neural networks trained through a hybrid approach comprised of a physics-based model and data-adaptive stage. The results indicate that the LSTM architecture is an improved predictor of the LAMP time-series maxima and first-order statistics compared with SimpleCode.

In practice, an entire matrix of condition combinations would be run through the seakeeping software to determine an operating envelope for each sea state. To account for the many combinations of conditions, especially in the early design stage, to obtain accurate but rapid estimates of these statistics is vital. The LSTM method provides a basis for addressing this problem in reducing the time to produce many realizations of different conditions quickly with a level of fidelity approaching a higher-fidelity code like LAMP. Of course, to be feasible, the method must demonstrate extensibility to other sea states, relative wave headings, and ship speeds to effectively reduce the computational effort. Still, even with training and testing on a single environmental and operating condition, the LSTM method could produce many higher-fidelity realizations in a relatively short period of time, which would be valuable for estimating extreme characteristics.

Based on the results of this study, the two-stage LSTM architecture trained to correct SimpleCode global positioning is a suitable candidate for further investigation and application to extreme event predictions.

Potential future work includes:

- For prediction structural loads, accelerations, and resistance.
- Extending assessment to cover a range of wave parameters including significant wave heights, modal periods, ship speeds, and relative wave directions.
- Application to other hull form geometries.
- Evaluation of LSTM network configurations in terms of hyperparameters and prediction performance.
- Investigation into Bayesian LSTM networks to include uncertainty in time series predictions

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CONTRIBUTION STATEMENT

Samuel J. Edwards: Conceptualization; software; data curation; methodology; formal analysis; visualization; writing – original draft. **Michael Levine:** conceptualization; writing – review and editing.

DATA ACCESS STATEMENT

No publicly accessible dataset was used in this paper.

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