A novel usage of rough sets in design of data fusion systems

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ABSTRACT

Design changes for crewless vessels are unexplored compared to the maritime design processes that have been utilized and updated for hundreds of years. This paper presents an exploration into how autonomous and unmanned systems can impact maritime design, specifically focusing on how well they can fuse multiple types of information. Currently, formal and informal communication onboard crewed vessels between various departments is critical in constructing a view of the vessel's current health and future capability. A major focus area is determining whether utilizing data classification techniques can replace these humancentered decision processes, and what the design implications of losing the human synthesis will be. This paper proposes a mechanical spring-mass-damper system with base excitation using real-world ocean data to be used to perform analyses. Rough Set Theory (RST) is a data classification technique that can be used for the characterization of a set of objects, finding dependency between attributes, and creating rules for making decisions. RST is compared with other data classification techniques to determine where each classifier succeeds and how they can generate information useful in design. By integrating the results of these analyses, this paper identifies ways to begin fusing multiple information types and how this will impact marine design in the future of crewless systems.

KEY WORDS

Digital Twin; Machine Learning; Design Uncertainty; Wave Forecast; Safety Prediction

INTRODUCTION

As human safety becomes increasingly more important in every aspect of life, the desire for crewless platforms grows in the ground, air, and marine domains. However, the necessity for operating crewless platforms for weeks to months is a challenging endeavor that separates the naval world from many other domains using autonomous or human-less systems. Designers currently lack guidance on which types of systems may be successful for these long-term applications. While a range of machine learning approaches have been proposed in the computer science literature, it is not clear how these methods could help the overall design process. This work compares a Rough Sets based approach to two conventional classifiers, looking at both accuracy and how the classifiers generate design-relevant information.

Collette et al. (2022) interviewed human crew members who had served or were serving on several types of ships and were involved in different roles both on and off the vessel. Crew members discussed that they were still deeply involved with making sure ship systems were healthy and functional even though there are preventative maintenance systems and sensors

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onboard that work without humans constantly in the loop. Through years of experience, humans can sense when or where there may be an issue even if it is not noted by a fault sensor or other component. Therefore, a major issue with crewless platforms is exposed – how will we be able to predict failure? This is imperative both in the design phase of a vessel and across the vessel's lifespan.

Remaining Useful Life (RUL) is an estimate of how long an item, component, or system can operate and fulfill its intended purpose before repair or replacement becomes necessary. Gebraeel et al. (2004) developed neural network models around bearing parameters and a parameter-updating algorithm that computed bearing failure time predictions. Liao et al. (2006) present a proportional hazards model and a logistic regression model in predicting the RUL of an individual unit. They also use a bearing test to demonstrate their proposed approach. Li et al. (2018) proposed a data-driven approach for prognostics using deep convolution neural networks. They then used their approach to perform and cross-compare an experimental study with NASA's C-MAPSS Dataset with other approaches. Cai et al. (2020) contributes a hybrid physics-model-based and data-driven remaining useful life (RUL) estimation methodology of structure systems considering the influence of multiple causes by using dynamic Bayesian networks. Finally, Aivaliotis et al. (2017) present an approach to try and provide a satisfactory solution for calculating RUL of machines in a production plant through PHM technique, leading to the idea of using a Digital Twin (DT).

A digital twin is a dynamic virtual representation of a physical object, person, system, or process that absorbs data and replicates processes to predict possible performance outcomes and issues. It will last the entire life of its physical twin, can be updated from real-time data, and can aid in the decision-making process of its twin. Kritzinger et al. (2018) provided a thorough review of DT in manufacturing at that time and showed that development was still relatively new but increasing in effort. Zhao et al. (2020) described a modeling method using DT for a manufacturing process, proposing a hierarchical model and mapping strategy for generating DT data, and Liu et al. (2023) proposed an updating method for DT knowledge based on a memorizing-forgetting model. On the maritime side, Raza et al. (2022) conducted research for applying DTs for autonomous vessels, and stated that DTs can be used to optimize path planning models using real world data such as sea charts and wave disturbance estimates. Mauro and Kana (2023) review the present-day status of DT research, and they state that the shipping industry is a few years delayed compared to other industry sectors, especially manufacturing. Finally, Kinaci (2023) discusses the need for DT for full autonomy in the seas, using a maneuvering math model to represent a physical ship. They developed a digital twin environment for a model ship and tested a control algorithm for sailing automation. Collectively, the prior research on digital twins contains several component examples, but digital twins are not discussed thoroughly from a design perspective, especially regarding autonomous vessels.

Failure prediction will be one of the most important aspects of autonomous vessels, especially in the design phase. These are platforms that do not have the experience of human crews and do not have enough underway data to learn from themselves yet. This work outlines a proposed system model which can be used to model component degradation. Several different classifiers are used on data generated from the model to predict failures and are compared with each other. Finally, rough sets was used to classify and analyze the data to be looked at from a design perspective. Background information on rough sets is included in the machine learning classifiers subsection of the next section.

SYSTEM SETUP AND CLASSIFIERS

Studying vessels designed for long-term autonomy is challenging, as full-scale prototypes are only in the early stages of development. Based on the literature review above, a model system was created to stand in for the autonomous vessel. The model system should have multiple interacting components whose health could be assessed in differing ways and a single output parameter to stand in for the capability of the overall vessel. The model system should also allow realistic weather forecasts and uncertainty to be included in the system. After some discussion, a spring-mass-damper system, excited by stochastic wave systems, was selected. The spring, mass, and damper can be modeled as degrading components; as the value for each change, the overall response of the system also changes. By setting a maximum allowed displacement on the system, an equivalent of a safety threshold, dependent on the health of all three components and the weather prediction, can be modeled. This section introduces the spring-mass-damper system, ocean wave parameters, and the overall setup of the simulation. The parameters used for each trial are described, and the machine learning classifiers are also introduced and detailed in this section.

Spring-Mass-Damper System

A spring-mass-damper system was developed to model component degradation and is composed of a mass block, a stiffness component, and a damping component, as well as using an ocean wave input as base displacement. The system can use different input parameters such as significant wave height and peak frequency to excite the dynamic system and track its response. Figure 1 is a physical drawing of the system. In the figure, *y* represents the base displacement, *x* represents the mass displacement, *m* is the mass component, *k* is the stiffness component, and *c* is the damping component.



Figure 1: Spring-Mass-Damper system diagram

Simulations using Ocean Weather Data

Using the correct system model and setup is critical for accurate, reusable, and scalable results. The weather data, i.e. the significant wave heights and peak periods used to model the base excitation, were taken from the European Centre for Medium-Range Weather Forecasts (©2023 European Centre for Medium-Range Weather Forecasts (ECMWF) ECMWF (2023)). This data is published under a Creative Commons Attribution 4.0 International (CC BY 4.0).

The ECMWF has open weather data containing wave data from across the Earth, and the ECMWF provides "global forecasts, climate reanalyses, and specific datasets". The data used for this project is five predictions for one specific time on one specific date, which was the 00:00 hour on May 22, 2023. Data was collected at 1000 points distributed across the world's oceans in order to collect a range of different significant wave heights and peak periods. The five different datasets are the real-time data from May 22, and weather predictions from 24, 72, 144, and 240 hours out. Figure 2 displays the significant wave heights at the 00:00 hour on May 22. The regions on the map filled with black markers are the coordinates where weather data was collected for this project.

The mass and stiffness values were chosen to generate a certain range of natural frequencies, which can be converted to natural periods. The periods were selected to correlate with the peak periods seen in the ocean data. The mass, stiffness, and damping values were generated using Latin Hypercube Sampling (LHS) with three dimensions and 1000 data points for each component. LHS distributed m, k, and c values between the selected minimum and maximum for each component, which are shown below in Table 1. The Bretschneider ocean wave spectrum was used to create the wave spectra used for the base excitation. Equation 1 displays the equation used for the Bretschneider wave spectra. This equation comes from the textbook *Offshore Hydrodynamics* (Journée (2001)).



Figure 2: Significant wave heights for 00:00 hour on May 22, 2023 with data collection regions overlaid

Table 1: Minimum and Maximum values for mass, stiffness, and damping components

	Mass (kg)	Stiffness (N/m)	Damping (Ns/m)
Minimum	1	1	0.1
Maximum	10	9	9.5

$$S_{\zeta}(\omega) = \frac{173 \cdot H_{\frac{1}{3}}^{2}}{T_{1}^{4}} \cdot \omega^{-5} \cdot exp(\frac{-692}{T_{1}^{4}} \cdot \omega^{-4})$$
(1)

$$T_1 = 0.772 \cdot T_p \tag{2}$$

Using the mass, stiffness, damping, and wave data values, the average maximum displacement of the mass was calculated for each of the 1000 data points. Equation 3 shows the amplitude for the vibration of a base-excited spring-mass system, and then, using the square root of the moment, multiplied by 3.85, the greatest response amplitude expected on 1000 independent observations is generated. An arbitrary threshold value was selected such that a mix of maximum displacements both exceeded the threshold and remained below the threshold. If the maximum displacement exceeded the threshold, the trial result was considered a 'failure', and if it remained below the threshold, the result was a 'pass'.

$$x = \frac{Ky(1 + (2\zeta\omega/\omega_n)^2)^{1/2}}{((1 - \omega^2/\omega_n^2)^2 + (2\zeta\omega/\omega_n^2))^{1/2}}$$
(3)

The first set of trials looked at how failure prediction accuracy changed with decreasing weather forecast accuracy. For these five trials, the exact m, k, and c values from the LHS results were used. For the base excitation, all 5 weather forecasts were used to determine how prediction accuracy is affected between the real-time zero-hour data and the 240-hour predicted weather data.

Simulations using Alternate Parameters

A major part of the Collette et al. (2022) report revolved around the human interviews with crew members. They reported that human crew members onboard marine vessels are deeply involved in the preventative maintenance process to the point that they are sometimes able to notice a fault or failure before a sensor or machine can report it. However, especially in the design space of autonomous vessels, human inspection will not exist and sensors will be necessary for sensing component health. This puts on the designers the need to choose sensors, sensor accuracy, and decision methods that will use the sensor data. However, very little guidance exists in this space right now.

Therefore, an additional column was added to the data table utilizing a 'visually inspected' spring. The stiffness values were taken from the LHS generations, and using three levels of accuracy, given a label. Results were labeled 'healthy', where the spring is in a healthy and optimal state, 'worn', where the spring is well-used but still functional, and might not work as optimal as in the 'healthy' state, and 'failed', where the spring is no longer operational. For a crewless vessel, this could correlate with an onboard imaging system or periodic human inspection between voyages. A trial was conducted using the visually inspected stiffness results, the same LHS results for mass and damping, and the zero-hour weather data to see how decreasing stiffness component knowledge accuracy affected the prediction accuracy.

Machine Learning Classifiers

Ultimately, a crewless vessel will need to be able to make its own assessment of safety, given its current understanding of its health and the forecasted weather. Collette et al. (2022) noted that this process was highly human-dependent at the moment, with extensive discussion among senior crew members used to arrive at integrated, vessel-level assessments. In designing a crewless vessel, this process will need to be automated. As a first approach, a machine learning classifier could be used to integrate component health information and weather forecasts to assess if the system is safe. In this work, three machine learning data classification techniques were applied to these test cases. Support vector machines (SVM) are supervised learning methods used for classification and regression, and this work features SVM from scikit-learn in Python with the default SVC method (Pedregosa et al. (2011)). An advantage of using SVM is that it uses a subset of training points in the decision function (called support vectors), so it is also memory efficient. SVMs are adaptable and can manage highdimensional data and nonlinear relationships, however, they are black-box models and the method of finding the final product may not always be understood. In Adadi and Berrada (2018), the authors discuss how black-box models do not disclose anything about internal design or structure, meaning users are finding results without thorough knowledge of how the model is working. On the other hand, glass-box models are completely exposed to the user from start to finish, so users have full knowledge of how the model works.

The second technique is a Decision Tree, which is a glass-box supervised learning method that makes predictions by learning simple decision rules from data features and previous data. A decision tree is a flowchart-like tree structure where internal nodes denote features, branches denote rules, and leaf nodes denote the results of the algorithm. This work uses the tree.DecisionTreeClassifier from sklearn to predict outcomes. It is used for both classification and regression problems, and because decision trees are glass-box models, they present options that allow for informed decisions to be made. For the design phase of the vessel, such rules are useful as they indicate to the designer which parameters and values are most important to understand. However, decision trees can be less accurate than SVM and other black-box models, depending on the problem.

Finally, Rough Set Theory (RST) was also compared. RST can be used for both classification and to give a deeper understanding of the problem and decision space, which is attractive for use in a design setting. RST was orginally proposed by Polish scientist Zdzislaw Pawlak and is a technique for identifying and learning on common patterns in data, especially in the case of uncertain and incomplete data (Pawlak (1982)). The mathematical foundations of this method are based on the set approximation of the classification space. The theory allows for a description of objects to be used which can contain information about various features. The precision, or amount of detail in the description can vary based on the knowledge of the feature and other limitations. RST uses indiscernibility relations to recognize and discover attribute relations. Therefore, it needs data to be in a discretized form to properly function, so the input data for rough sets was sorted into bins for the data analysis and performance accuracy test.

Also in RST, two approximations are developed: The upper approximation is the set of objects which possibly but do not definitely belong to the target set, and the lower approximation is the set of objects which positively belong to the target set. Objects that fall between the lower and upper approximation are said to be in the boundary region. If the boundary region is non-empty, the set is said to be rough, otherwise it is a crisp set and all cases can be classified without fail.

Large volumes of data can be difficult to classify into specific categories through visual inspection, and RST can discern and classify objects in large data sets. RST uses the indiscernibility relations and lower and upper approximations to characterize and express an information system, and therefore does not require additional parameters to extract information. Rough Sets results were generated in this paper using the RoughSets package in R (Janusz et al. (2020)). The process for performing the data analysis in R begins with taking in the dataframe with the simulation results and converting it into a decision table. Then, the indiscernibility relation is computed and utilized with the decision table to compute the lower and upper approximations. Like with the SVM and decision tree classifiers, the data was split into training and testing sets, and the classifier was then able to compute the predictions for the test set, determine the overall accuracy of the predictions, and generate a list of rules.

Assessment Approach

From the 1000 data points, 700 were used for training the classifiers and 300 were used for the testing phase. The overall accuracy of the predictions was the main focus point for each classifier in comparing them to each other and is defined as the percentage of test trials where the predicted outcome matches the true outcome. The results and comparisons between classifiers are given in the following sections.

RESULTS AND DISCUSSION

The results from the simulations were used with the previously described data classification approaches to predict possible failures. Two different approaches were simulated, the first being the case where the mass, stiffness, and damping values all come from the Latin Hypercube Sampling. These are run with each weather data-set out to 240 hours to see how each classifier handles the changing weather uncertainties. The results are shown in Table 2, with the classifiers bolded in each row, the trial bolded in each column, and the table filled out with the accuracy percentages.

Fable 2:	Table of	prediction	accuracies (in	percentage)	for	each	classifi	ier in	each	trial	case
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	0-hour	24-hour	72-hour	144-hour	240-hour
SVM	95.28	95.28	94.85	93.06	90.66
Decision Tree	91.50	91.50	90.72	88.55	85.74
Rough Sets	91.95	91.95	89.70	86.30	84.20

The results show that overall, SVM outperforms the other two classifiers in prediction accuracy. Rough sets performs similar to but better than the decision tree for the 0 and 24 hour trials, but worse than the decision tree for the other three weather prediction cases. In the case of SVM and decision tree, the prediction accuracies remain similar from the 0 hour trial to the 72 hour trial, and then the accuracies drop significantly as the weather prediction becomes increasingly inaccurate. Rough sets performs similarly, except it loses its accuracy at 72 hours instead.

When thinking about how these results can be thought of in the design space, the drop off in accuracy as the weather knowledge weakens should be the main focus point. It emphasizes how designs need to focus on not only overall vessel safety, but also the unpredictability of the oceans. On crewed vessels, humans have the ability to make decisions based on their past experiences and knowledge of how well their ship can handle a certain weather obstacle. On uncrewed vessels however, the vessel itself may need to make the decision on whether to change its mission or remain on course. Being able to do this from 72 to 144 hours out is much more optimal than only having 24 hours to make a well-informed and accurate decision.

The second approach involves using the same mass and damping component values, but using a 'visually inspected' stiffness component to emulate the findings from the human crew interviews mentioned in Collette's paper. The prediction accuracy results for this approach are shown in Table 3 along with the initial result from the first trial.

 Table 3: Table of prediction accuracies (in percentage) for each classifier in the trial case with a visually inspected stiffness component compared to the original test case. The new trial case also uses the 0-hour weather data.

	0-hour	Visually Inspected
SVM	95.28	93.67
Decision Tree	91.50	90.17
Rough Sets	91.95	91.90

The 0-hour result from the first trial is included and compared to the visually inspected trial results because that case contains the most knowledge about the system. The results show that the SVM and decision tree results drop in accuracy by about 1.6 percent and 1.33 percent respectively from the 0-hour, full data case to the 0-hour, visually inspected stiffness component case. However, the rough sets prediction accuracy remains about level with only a drop of 0.05 percent.

The decrease in accuracy is expected with less knowledge about the system as a whole, but rough sets holding its accuracy was unexpected. Again, rough sets prove to be better than the decision tree at this stage and slightly worse than SVM, but closer than before. Rough sets also provides other options for data analysis as opposed to SVM and decision tree, which is detailed in the following section.

IMPACT ON DESIGN

While Rough Sets may not have outperformed the SVM and decision tree classifiers in terms of failure prediction accuracy, it also provides other information that can be more useful in design. The boundary region was defined as being the area between the lower and upper approximation, where the lower approximation was the set of objects that positively belonged to the target set.

The accuracy results show that from the real-time data to the 72 hour prediction data, there is not much of a drop in accuracy. However, there is a significant drop in prediction accuracy when moving to the 144 hour weather prediction data. Therefore, we looked at how the boundary region changes while using Rough Sets from the 0 hour case to the 144 hour case. Six different bin sizes were used for each case, with the bin sizes meaning how knowledgeable the system is on component accuracy. The number of bins ranges from 3 to 20, where the number of bins means that the exact numeric values of mass, stiffness, and significant wave height data have been placed into the corresponding number of bins between their minimums and maximums to discretize them. Such binning is also related to the accuracy of potential sensors for each component.

The trial used for this test case is the visually inspected spring component, along with the bin values in place of the previous mass, damping, and significant wave height components. Table 4 shows the number of cases (out of 1000) that were placed in the boundary region for the different number of bins in each trial.

Table 4: Number of test cases in the boundary region for differing numbers of bins

# of Bins	3	5	7	10	15	20
0h Trial	542	261	143	29	4	8
144h Trial	609	303	150	45	8	10

Looking at the table, the number of bins and the number of cases in the boundary region are clearly related. With a lower

number of bins, such as 3 or 5, there are more cases in the boundary region than at 10 or 15 bins. This shows that the number of bins affects our knowledge accuracy. As we begin knowing more about the system and can better differentiate between data, we can place them more accurately where they belong.

This can be related to design in looking at things such as sensor fidelity. If a sensor can only offer 3 or 5 different knowledge states about a part or product, it might not be worth selecting over one that offers 10 knowledge states. This is especially relevant in autonomous vessels, where knowing the difference between a component being healthy, worn, or failed may make the difference between a successful mission and a lost or compromised asset. However, at the extreme right hand side of the table, a different story is shown. Here, there is little advantage in selecting a sensor accuracy corresponding to 20 bins over one with 15, at least in terms of how well the data can be classified into sets.

Also being compared was the 0 hour trial to the 144 hour trial. The immense fall-off in prediction accuracy for the 144 hour case allowed for analysis in this situation as well to see if a similar pattern developed. While there was not an exact pattern, the superior knowledge of the 0 hour trial outperformed the 144 hour trial at every number of bins. Some cases, such as 7 bins and 20 bins did not have much difference, but with 10 and 15 bins, the 144 hour trial had 66 percent more cases and twice as many cases in the boundary region, respectively. This difference shows how more accurate knowledge about one aspect of a system, in this case the weather component, can allow for better design choices to be made.

Being able to visualize the impact on classification via the size of the boundary region is an important advantage in using a system like rough sets. Compared to black-box classifiers, whose performance may change with hyperparameter settings and the pipeline used to build the model, the boundary region is a fixed property of the engineering data set used to describe the problem. It provides a second way of looking at the potential for classifiers to work on the base problem that is quickly humanly understandable. Differing sensor accuracy and real-world uncertainty can also be examined.

CONCLUSION

This paper presented a system that was used to model component degradation and knowledge degradation. The springmass-damper system was combined with real-world ocean data to excite the system and 1000 test cases were simulated. Different machine learning classifiers were used to predict operational successes and failures and were compared to each other based on their accuracies. Support vector machines (SVM) was the best classifier in the sense of prediction accuracy, as compared to decision tree and rough sets. However, Rough Sets provided opportunities to look at the system in different ways, specifically how we can use it to think about design.

Rough sets have the ability to not only predict results from learning on training data, they also can perform other operations. Here, rough sets was used to look into how changing knowledge accuracy about a system or component can affect design knowledge. We saw that as we increased our sensor fidelity, the number of cases that could be positively classified increased immensely. This has design implications where rough sets can be used to determine which configurations or components can be considered in an optimal design region, or if they might need to be worked on further to narrow down the feasible design space.

CONTRIBUTION STATEMENT

Brendan Sulkowski: Conceptualization; Investigation; Data curation; Software; Methodology; Writing – original draft. **Matthew Collette:** Conceptualization; Writing – review and editing; Supervision; Funding.

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