

Age of Air Measurement in Air-recirculating Systems Applying Dynamic Steady-state Concentration Theory

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Abstract. In open air systems which is rooms with ventilation by fresh air, the evaluation of an indoor air environment based on age of air measurement, such as a stepdown method using tracer gas, is widely applied. However, in a ventilation room with air-recirculating systems such as multiple packaged air conditioning unit systems, the concentration continues to increase endlessly when there is no ventilation with fresh air. Thus, it is not possible to measure the air age using the conventional experimental method, as noted. Therefore, in previous studies, a tracer gas experiment was established by applying a dynamic steady-state concentration theory. The dynamic steady-state concentration is the concentration transition of each single source, assuming that the steady-state concentration is composed of the difference between the room source and the recirculating source. Previous studies have confirmed that the dynamic steady-state concentration agrees with the equivalent steady-state concentration in the ventilation room of an open air system. In addition, simulations by computational fluid dynamics (CFD) analysis and tracer gas experiments in the laboratory showed that the dynamic steady-state concentration of only the recirculating part of the source equals the age of air. With all these methods and studies, we attempted to extend to a real space with air-recirculating systems using the tracer gas experimental method based on dynamic steady concentration. First, we measured the air age in a real space with multiple packaged air-conditioning unit systems. We thereafter confirmed the distribution property of the multiple packaged air conditioning unit systems and verified the validity of the quantitative evaluation using CFD analysis. Our findings were as follows:

1. The age of the air measurement in a room with two or more recirculating parts was clarified by applying the dynamic steady-state concentration theory.

2. The tracer gas experiment of the dynamic steady-state concentration enabled the quantitative evaluation of the air distribution characteristics of the multiple-packaged air-conditioning unit system in real space.

Keywords. age of air, air-recirculating systems, dynamic steady-state concentration, multiple packaged air conditioning unit systems, tracer gas experiments

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1. Introduction

In ventilated rooms, tracer gas has been used to evaluate fresh air distribution properties and polluted air emission efficiency. With the introduction of the concept of "the age of air" by Sandburg et al.[1][2], pulse, step-up, and step-down measurement methods have been proposed, and measurement devices have been developed[3]-[6]. However, although experiments with tracer gas can be performed in open air systems which is rooms with ventilation by fresh air, experimental methods for air-recirculating systems where exhausted air is recirculated. have not been established. For example, dust collectors in smoking rooms, air purifiers in rooms with drifting pollen and house dust, and air conditioners without ventilation are in high demand to investigate these air conditions.

Thus, we developed a method to measure the concentration distribution of pollutants and the age of air in a room by tracer gas experiments in air-recirculating systems.[7] In addition, owing to COVID19, it is strongly desirable to be able to quantitatively evaluate the effect of improving indoor air quality in air-recirculating systems.

In this study, using the dynamic steadystate which will be described later, we understand the air distribution of two air conditioners (hereinafter, for simplicity, multiple packaged air conditioning unit systems are referred to as air conditioners.). The purpose of this study is to confirm the validity of the results by comparing them with computational fluid dynamics (CFD) analysis.

2. Dynamic steady-state concentration theory

2.1 Dynamic steady-state concentration theory with perfect mixing condition

Air-recirculating systems with leakages have room source M, volume V, airflow Q, and leakage air flow q, as shown in Fig. 1(a). Air-recirculating systems have a recirculating part source βM , as shown in Fig. 1(b), where β denotes the room source removal efficiency. For simplicity, the concentration of the leakage (in) was assumed to be zero in both cases. Assuming that room concentration is a perfect mixing condition, the concentrations of leakage (out) are C_P and C_N . Substituting Equation (2) from Equation (1), we obtain Equation (3). This is illustrated in Fig. 1(c).



Fig. 1- Tracer gas experimental method with leakage

We subtracted the concentration in Fig. 1(b) from that in Fig. 1(a) according to β at the right time. Equation (4) is established, and the right side of Equation (3) becomes zero. Under these conditions, $C_P - C_N$ becomes a steady-state concentration. This is shown in Fig. 2. Equations (1) and (2) change with time until they reach the steady-state concentration. However, $C_P - C_N$ remained constant at $(1 - \beta)M/q$. The steady-state concentration in the open air system is represented by the difference between the room source and the recirculating part source of the air-recirculating system. The timevarying concentration transition of the room source and recirculating part source, which constitutes the steady-state concentration distribution, is known as the dynamic steady-state concentration.



Fig. 2 – Dynamic steady-state concentration transition

$$V \frac{dC_P}{dt} = M - C_P q \quad \dots (1) \quad V \frac{dC_N}{dt} = \beta M - C_N q \quad \dots (2)$$
$$V \frac{d(C_P - C_N)}{dt} = (1 - \beta)M - (C_P - C_N)q \quad \dots (3)$$
$$C_P - C_N = (1 - \beta)\frac{M}{q} \quad \dots (4)$$

2.2 Dynamic steady-state concentration theory with non-perfect mixing condition

Next, we consider the case with non-perfect mixing condition. A block model of the air-recirculating systems divided by microvolume elements is shown in Fig. 3, where the air flow from element *i* to *j* is Q_{ij} , room source is $m_{p,i}$, recirculating part source is $m_{n,i}$, leakage air flow from element *i* to the outside is q_i , and concentrations are $C_{p,i}$ and $C_{n,i}$, respectively.



Fig. 3 – Block model of the air-recirculating systems divided by micro volume elements

Their concentration changes are shown in Equations (5) and (6), respectively. The sum of $m_{p,i}$ is *M* and that of $m_{n,i}$ is βM . The average concentrations were C_P and C_N , and the average concentrations (room to outside by leakage) were $C_{P,ex}$ and $C_{N,ex}$. When Equations (5) and (6) were summed over the entire space, they were obtained from Equations (1) and (2), respectively. If Equation (4) is satisfied, the spatial average concentration difference, C_P - C_N , does not change with time. We assume α_i is independent of the source generation points and is determined by each point in space. If Equations (7) and (8) are satisfied, then $C_{P,i} - C_{N,i}$ is a steady-state concentration. To achieve this, the average concentration must be approximated by Equation (9), using T = V/q. Under this condition, by transforming Equation (5) using Equations (7) and (9), α_i becomes the solution of Equations (10) and

(11). In particular, without leakage, $q_i = 0$ and $T \rightarrow \infty$; therefore, the solution to Equations (10) and (11) is $\alpha_i = 1$ for all *i*. This implies that the rate of change in concentration with time is constant at each point in space. Equation (11) becomes $\alpha_i \approx 1$ because $V = \sum v_i$ is valid. Therefore, we can expect $\alpha_i \approx 1$ under the condition that q << Q.

$$\begin{aligned} v_{i} \frac{dC_{P,i}}{dt} &= \sum_{j} C_{P,j} Q_{ji} - \sum_{j} C_{P,i} Q_{ij} + m_{P,i} - C_{P,i} q_{i} & \cdots (5) \\ v_{i} \frac{dC_{N,i}}{dt} &= \sum_{j} C_{N,j} Q_{ji} - \sum_{j} C_{N,i} Q_{ij} + m_{N,i} - C_{N,i} q_{i} & \cdots (6) \\ \frac{dC_{P,i}}{dt} &= \alpha_{i} \frac{dC_{P}}{dt} \cdots (7) & \frac{dC_{N,i}}{dt} = \alpha_{i} \frac{dC_{N}}{dt} \cdots (8) \\ C_{P} &= C_{P0} e^{-t/T} + C_{\infty} (1 - e^{-t/T}) & \cdots (9) \\ \left(\sum_{j} Q_{ij} + q_{i} - \frac{v_{i}}{T}\right) \alpha_{i} - \sum_{j} Q_{ji} \alpha_{j} = 0 & \cdots (10) \\ V &= \sum_{i} v_{i} \alpha_{i} & \cdots (11) \end{aligned}$$

2.3 Relation between dynamic steady-state concentration and age of air

This is the block model of the airrecirculating part source, as shown in Fig. 4.



Fig. 4 - Block model of the air-recirculating systems divided by micro volume elements (recirculating part source)

Without leakage, Equation (6) becomes Equation (12). Because $\alpha_i = 1$, if $\beta = 1$, the change in concentration is constant, which becomes Equation (13). The sum of the air inflow and outflow for each microvolume element was $\sum_i Q_{ij} = \sum_i Q_{ji}$. Therefore, Equation (13) becomes Equation (14). C_0 is an arbitrary value. The age of air is $\tau_i = C_0 - C_{N_j}$, which is the steady-state concentration in the room under $M \propto V$ conditions. The term C_0 refers to the correction for setting the age of air in the supply air to zero.

With leakage, Equation (14) becomes Equation (15). When a tracer step-up experiment is conducted in air-recirculating systems with leakage, the recirculating part source is consumed by the increase in the concentration and leakage, as expressed in Equation (2). The term γ represents the fraction spent on increasing the concentration, which was $1 \rightarrow 0$. Therefore, under q << Q and just after the start of the experiment, Equation (15) is nearly equal to Equation (14); therefore, the

dynamic steady-state concentration is expected to approximate the age of air.

$$\begin{aligned} v_{i} \frac{dC_{N,i}}{dt} &= \sum_{j} C_{N,j} Q_{ji} - \sum_{j} C_{N,i} Q_{ij} + m_{N,i} \quad \cdots (12) \\ 0 &= \sum_{j} C_{N,j} Q_{ji} - \sum_{j} C_{N,i} Q_{ij} + m_{N,i} - v_{i} \frac{M}{V} \quad \cdots (13) \\ 0 &= \sum_{j} (C_{0} - C_{N,j}) Q_{ji} - \sum_{j} (C_{0} - C_{N,i}) Q_{ij} - m_{N,i} + \frac{v_{i}}{V} M \quad \cdots (14) \\ 0 &= \sum_{j} (C_{0} - C_{N,j}) Q_{ji} - \sum_{j} (C_{0} - C_{N,i}) Q_{ij} - m_{N,i} + C_{0} p_{i} \\ - (C_{0} - C_{N,i}) q_{i} + \gamma \alpha_{i} \frac{v_{i}}{V} M \qquad \cdots (15) \end{aligned}$$

2.4 Measuring method of age of air with multiple recirculating part sources by dynamic steady-state concentration

Fig. 5(a) and (b) shows the situation after the correction. Because the concentration transition is linear, the concentration distributions in (a) and (b) are multiplied by correction factors k1 and k2, respectively. Figure 5(c) shows the situation in which they were synthesized. Equation (16) is the condition for setting the blowing concentration to zero, and Equation (17) is the condition for the total source volume to be equal to the source volume V (in a room under M \propto V conditions). Equations (16) and (17) are quadratic equations with k1 and k2 as unknowns, and we can obtain k1,k2 using these equations. Figure 5(d) shows the situation after the correction of Fig. 5(c).



Fig. 5 - Correction method with two recirculating parts

3. Experiment

3.1 Outline of experiment

Fig. 6(a) and (b) show the plan and A-A' sectional views of the target room and positions of the air conditioners. We placed 4×4 highly responsive CO₂ concentration measuring instruments on the same

plane (h = 700). Tracer gases were generated by air conditioners (plan view painted in red or blue). Figure 6(c) shows the details of the air conditioners.



Fig. 6 - Target room overview and detail of air conditioners

Fig. 7 shows the results of the airtightness measurement and the allowable leakage time. We perfromed a concentration decay method using tracer gas in the target room. The ventilation rate was used as the leakage rate in the target room, and the leakage rate was 0.0233 times/h. Equation (18) expresses the change in concentration in a room with ventiration rate n in an open air system. The rate of increase in concentration is obtained from Equation (19). If we allow a 5% error, we must use data up to nt = 0.05, implying that the measurement limit in the target room is approximately 2.1 h. Tab.1 lists the case studies. The four cases depend on the generation point and airflow.



Fig. 7 – Airtightness measurement results

$C = \frac{M}{q} \{1 - \exp(-nt)\} \cdots (18)$	
$V\frac{dC}{dt} = V\frac{d}{dt}\left\{-\frac{M}{q}\exp(-nt)\right\} = V\frac{M}{q}\frac{q}{V}\exp(-nt) = M\cdot\exp(-nt) \cdots$	(19)
$V \frac{dC}{dt} = M \cdots (20)$	

Tab. 1 - Study case

Case	Tracer Gas Generation	Airflow [m³/(h*one)]	Blowing Angle
Case A ₁	Red	(12	200
Case A ₂	Blue	612	30°
Case B_1	Red	1120	200
Case B ₂	Blue	1138	30*

3.2 Experimental result

Fig. - 8 shows the concentration transition of the four cases and the sampling time. The previous studymust measure the air age at approximately 1/2 of the nominal ventilation time (V/Q) after the formation of the concentration distribution[8]. However, in case A_2 , we measured the concentration at the time when it was stable because the concentration was not stable at the measurement points K, L, O, and P.







Fig. 8 - Concentration transition of the four cases

Fig. 9 shows the results for the four cases. The values are the SVE3 of each concentration, with the blowing concentration corrected to zero. In all cases, the age of the air near the generation point is better, and the age of the air becomes worse as it moves away from the generation point. When comparing cases A_1 and B_1 , the values were generally the same. However, when comparing A_2 and B_2 , the values near the generation location are the same, but B_2 is better at a distance from the generation location. In case A_2 , the concentration transition near the generation location (K, L, O, and P) was not stable, but the values in A_2 and B_2 were generally the same; therefore, the results were not affected.

Fig. 10(a) and (b) shows the synthesis results for A_1 and A_2 , and B_1 and B_2 , respectively. Comparing (a) and (b), the distributions are almost the same and the value of (b) is better than that of (a). We confirmed a difference in the range of forces depending on airflow.



Fig. 10 – Synthesized results : age of air

(b) caseB₁+B₂

4. CFD analysis

(a) caseA1+A2

4.1 Outline of CFD analysis

Tab. 2 lists the analytical conditions, and Fig. 11 shows the analytical model. The age of air in the airrecirculating systems is equal to the age of air in the equivalent open air system. A steady-state analysis was performed with two air conditioners as the source locations under open conditions.

Tab. 2 – CFD	analysis	conditions
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Software	STAR-CCM+(2020.3)
Analysis Area[m³]	13.54(x)*13.13(y)*3.015(z)
Total Mesh Size	1,503,668
Turbulence Model	Standard k-εModel
Airflow Analysis	2nd-order upwind scheme
Passive Scalar Analysis	1st-order upwind scheme



Fig. 11 – CFD analysis model

Fig. 12 shows the CFD results and Fig. 13 shows the correlation between the CFD results and the experiment. Comparing Fig. 12(a) and Fig.10(a), the values in Fig. 12(a) in the upper right corner of the room are higher than those in Fig. 10(a), but the values near the generation point are approximately the same. Although the distributions were generally consistent, there was a slight difference in the RMSE = 0.154. Fig. 12(b) and the experimental data exhibit a similar trend, dispite *RMSE* = 0.109



Fig. 13 - Correlation between the CFD results and the experiment

The problem is that the only way to measure the airblowing angle of the air conditioner is by visual measurement, and case studies are conducted by changing the angle in the analysis. Therefore, a future task is to improve the accuracy of the analysis of air conditioners with oblique blowing angles.

5. Conclusion

1. The age of air measurement in a room with two or more recirculating parts was clarified by applying the dynamic steadystate concentration theory.

2. The tracer gas experiment of the dynamic steady-

state concentration enabled the quantitative evaluation of the air distribution characteristics of the multiple-packaged air-conditioning unit in a real space.

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c.The datasets generated during and/or analysed during the current study are not available because authers take time to organeze the data but the authors will make every reasonable effort to publish them in near future.