A Normal Form for Representing Legal Norms and its Visualisation Through Normative Diagrams.

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Abstract. Representing legal norms by means of an adequate formalism is essential for the development of legally compliant autonomous systems. The remarkable variety of both formalisation and implementation frameworks proposed during the last decades enables the development of relatively efficient formalisations over a wide range of applications. Notwithstanding, this abundance of frameworks also leads to some practical problems, especially with respect to the integration of – or the translation between – different frameworks. While more or less widely applicable meta-formalisms are already available, they are often too complex and counter-intuitive, thus being clearly closer to the technical implementation than to the conception phase of the formalisation process. As a complementary, more human-oriented solution to the problem, the paper at hand introduces and discusses a more intuitive normal form structure, which is based on elements of legal theory and aims at facilitating the interdisciplinary communication among the many partners involved in the process of developing law-abiding machines.

Keywords. Legal knowledge representation, formalisation of legal norms, deontic logic, legal logic, normal form, normative diagrams, autonomous driving, legal visualisation
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1. Introduction

Current advancements in the field of artificial intelligence are paving the way for the integration of autonomous systems in various areas of human activity. Autonomous agents can recommend products, steer vehicles or assist in solving difficult formal problems. Within public administration, such systems can be employed to promote the automatisation of public services, to assist public authorities in making decisions under their jurisdiction, and enhance the administration of public goods in general (cf., e.g., Dolata and Schwabe, 2023; Döring et al., 2024; Larsen and Følstad, 2024; Lindgren, 2024). In the realm of law, autonomous systems can also be employed to provide more accurate information concerning the legal aspects of specific cases – thus promoting a better access to justice –, to assist judges, lawyers and other legal practitioners in their work or also to facilitate online dispute resolution (cf., e.g., Carneiro et al., 2014; Raabe et al., 2012; Virkar et al., 2022). Whilst the main objective of these systems is to be effective and efficient in solving the respectively given problems, especially when they are supposed to directly interact with humans, they should also behave in a law-abiding manner. Thus, beyond being merely effective and efficient, autonomous systems should also be lawful.

The challenges involved in achieving law-abiding autonomous systems has lead to the development of various eclectic logical formalisms and implementation frameworks (for example, *LegalRuleML*; cf. Palmirani et al., 2011). However, these frameworks are oftentimes designed from an engineer's point of view and are thus less intuitive to legal scholars and practitioners. This often leads to difficulties in the interdisciplinary task of formalising legal norms. To provide a method for the specification of these problems in a way intuitive for both engineers *and* legal professionals, the paper at hand presents a type of *normal form* for representing legal norms based on elements of legal theory. Starting with high-level descriptions, this normal form allows for a swift transition between various degrees of specification, thus facilitating the communication between actors in the many implementation layers involved when designing autonomous systems. A further advantage of the normal form presented here is the fact that it can be easily combined with so-called *normative diagrams*, thus also enabling the construction of intuitive visualisations for the logical formalae employed.

In the following, Sec. 2 gives a short introduction to the general task of formalising legal norms using an example from German traffic law and demonstrates the various problems that may arise in the formalisation process. Sec. 3 briefly discusses related work on existing widely applicable meta-formalisms for legal norms and on the field of legal visualisation. Sect. 4 then presents the here proposed normal form solution and discusses how its structure facilitates the employment of a geometric visualisation method for legal norms by means of so-called *normative diagrams*. These diagrams could be particularly useful, e.g., if integrated in an intuitive *legal norm formalisation tool* directed at supporting legal scholars when formalising norms. Sec. 5 illustrates the use of the normal form (and of normative diagrams) through the discussion of a detailed example. Finally, Sec. 6 concludes and points out further directions for future research.

2. Formalising Legal Norms as a Complex Interdisciplinary Task

The process of formalising legal norms consists of several steps and involves experts from different disciplines. The complexity and interdependencies of this process can be illustrated through the example of developing an autonomous vehicle (AV). Designing an AV that is capable of driving *safely* through a city – a rather challenging task for itself – is not enough. Beyond *safety* requirements, AVs also have to fulfil *legal* requirements, i.e., they have to comply with existing traffic rules. In fact, some countries have already passed legislation requiring AVs to be able to autonomously comply with traffic rules, e.g., Germany's *Straßenverkehrsgesetz* (*StVO*)in its § 1e (2).

In a nutshell, the formalisation task consists of implementing traffic rules for AVs so that they are able to obey them like humans do. But to autonomously comply with legal norms, these norms must first be adequately *represented*. In particular, the representation method employed should enable AVs to recognise (ideally in advance) which manoeuvres they could perform would constitute a violation of a given legal norm. Moreover, in more *difficult* cases, e.g., in so-called *contrary-to-duty* situations or in cases involving conflicting norms (in which the violation of at least one norm is unavoidable), AVs should (ideally) be able to adequately evaluate and choose those manoeuvres which constitute lesser violations. Thankfully, literature offers countless formalisms – e.g., different systems of (deontic) logic (Gabbay et al., 2013) – which promise to deliver a more or less accurate representation of the various types of reasoning involved in such scenarios. However, the task of developing law-abiding AVs is not solely a logic problem, and it neither begins nor ends with the choice of an adequate formalism. Consider, e.g., the situation illustrated in Fig. 1. The autonomous vehicle AV is driving in Germany on the road r, which is partially obstructed by some unidentified object $\mathfrak{O}$. AV could drive on by performing the manoeuvre m_1 that would lead it through the indicated trajectory. As an autonomous system, AV should be able to determine whether is is allowed to do so.

At first glance, the case seems to be regulated by § 6 of the German Road Traffic Regulation (*StVO*). § 6 regulates a manoeuvre-type called *passing* (*Vorbeifahren*). It states that drivers wishing to *pass* (because of a road narrowing, an obstacle on the carriageway or a stationary vehicle) by driving onto the left lane must allow oncoming vehicles to pass through. A simple way to represent this in first-order logic would be:

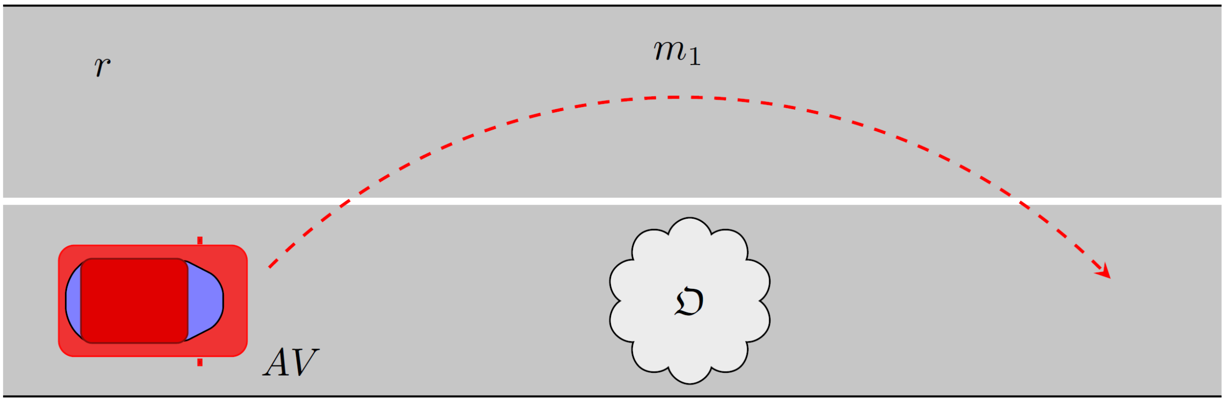


Fig. 1 – Would it be allowed for AV to drive past the unidentified object $\mathfrak{O}$ by taking the indicated trajectory?

$$\begin{aligned}&\forall x, y, z(\\&(Driver(x)\\&\wedge (Rd\_Narrowing(y) \vee Obstacle(y) \vee St\_Vehicle(y))\\&\wedge Onc\_Vehicle(z, x)\\&\wedge Passing(x, y)\\&\qquad\qquad) \rightarrow Must\_Allow\_Pass\_Through(x, z))\end{aligned} \quad (1)$$

Here, the predicate $Driver$ represents the property of being the operator of a vehicle (e.g., a human driver or an autonomous driving system). $Rd\_Narrowing$, $Obstacle$ and $St\_Vehicle$ are predicates corresponding to the conditions stated in § 6 StVO: They indicate that the driving situation is such that the lane in which one is driving is obstructed either by a road narrowing, an obstacle or a stationary vehicle. $Onc\_Vehicle$ corresponds to the property of being a vehicle driving on the opposite lane. $Passing$ corresponds to the fact that a driver intends to perform the passing manoeuvre as defined in § 6 StVO. Finally, $Must\_Allow\_Pass\_Through$ corresponds to the obligation that a driver intending to perform the passing manoeuvre has to first let oncoming traffic pass through. The variables x, y and z are assumed to be ranging over adequate sets of objects (e.g., drivers for x, road obstacles for y, vehicles for z).

Evidently, $Driver(AV)$ holds, and, at least intuitively, $Obstacle(\mathfrak{O})$ also seems to be satisfied. Hence, as long as there are no oncoming vehicles, it would seem that AV is allowed to execute the manoeuvrer m_1. In case there were oncoming vehicles, AV would only be allowed to perform m_1 if this were not to impede said vehicles from passing through, i.e., so as to satisfy $Must\_Allow\_Pass\_Through(AV, z)$, where z is an oncoming vehicle.

However, a more careful consideration of the example depicted in Fig. 1 reveals that the road r, on which AV is driving, bears a specific road marking, namely a ***solid line***. The legal norm connected to this road marking is described in the second annex to § 41 (1) of the StVO (*sign 295*). Following this norm, a vehicle driving on a road with a solid line must neither cross nor even straddle the line. A simple first-order formalisation of this norm would be:

$$\begin{aligned}&\forall x, y(\\&(Driver(x)\\&\wedge R\_Sld\_Line(y)\\&\wedge Drives\_On(x, y)\\&\qquad\qquad) \rightarrow (Must\_Not\_Cross\_Line(x)\\&\qquad\qquad \wedge Must\_Not\_Straddle\_Line(x)))\end{aligned} \quad (2)$$

Since $Driver(AV)$, $R\_Sld\_Line(r)$ (road has a solid line) and $Driver\_On(AV, r)$ hold, AV must neither cross nor straddle the solid line on r. Since, as implied by Fig. 1, performing m_1 requires crossing the solid line, the manoeuvre would actually be forbidden. This would mean that AV would have to wait behind $\mathfrak{O}$ until the carriageway is cleared or at least until it is able to pass by $\mathfrak{O}$ *without* crossing or straddling the line (Helle, 2021, Rn. 14). Formalisations (1) and (2) are, however, still far from ideal. Overall, they display two general problems, namely a *legal-theoretical* and a *technical* one.

The *legal-theoretical problem* consists in the fact that formalisations (1) and (2) do not really consider the valid *legal norms* applicable to the case, but only (some parts of) the corresponding *legal texts*. Formalisation proceeds from the assumption that law is a system of legal norms. From this perspective, it is important to distinguish between the law itself and its many sources, in particular between the law and written legislation (Nawiasky, 1948). Overall, legal norms are obtained from legal sources through the use of different interpretation techniques (Alexy, 1994, pp. 42-47, Weinberger & Weinberger, 1979, pp. 108-112). One can distinguish between five different classes of legal sources (for a more detailed discussion, cf., e.g., Kelsen, 2017, pp. 417-423, Bobbio, 1993, pp. 173-182): (I) written legislation; (II) legal precedents; (III) private autonomy (e.g., contracts, wills etc.); (IV) usages and customs; (V) legal dogmatics (e.g., the opinion of legal scholars). Hence, assessing the actual *norms* applicable to a given case requires way more than simply reading the text of written legislation. This is illustrated well by the fact that the last assessment given above, i.e., that AV would have to wait behind $\mathfrak{O}$ due to the solid line, is actually wrong. Following a well-established understanding adopted by German courts, a driver passing an obstacle in the sense of § 6 of the StVO is allowed to cross the solid line (Helle, 2021, Rn. 14; cf. also the court decisions OLG Saarbrücken, decision from 01.12.2016, 4 U 109/15, Rn. 47; OLG Düsseldorf, decision from 07.06.2022, I-1 U 222/20 Rn. 61). In other words, § 6 entails an (*unwritten*) exception to the solid line rule. The importance of interpretation becomes even clearer when considering the vagueness of the concepts used in § 6. For example, the predicate $Obstacle$, as represented in (1), could, intuitively, be satisfied *prima facie* by anything standing on the carriageway. Yet a more careful consideration of how legal dogmatics defines the concept of *passing* reveals that a non-stationary vehicle (or even a vehicle that stopped due to the traffic circumstances (Helle, 2021, Rn. 6, Heß, 2022a, Rn. 2-3) would not qualify as an obstacle in the sense of this norm. In other words, one does not *pass* by a non-stationary vehicle, one *overtakes* it. In the StVO, passing and overtaking (*Überholen*) are two different manoeuvres. Overtaking is regulated in § 5, and, more importantly for the case at hand, it does *not* constitute an exception to the solid line rule (Heß, 2022b, Rn. 92; cf. also the decision BGH NJW-RR 1987, 1048, 1049). Hence, whether or not AV is allowed to execute the manoeuvre m_1 depends on what exactly the object $\mathfrak{O}$ is.

This leads to the second, *technical problem* raised by formalisations (1) and (2): such formalisations have to be *implemented* in real machines, which leads to a series of challenges. For example, the predicates employed in formalisations of legal norms are often not logically independent from one another. For example, if r is a road with a solid line, i.e., if $R\_Sld\_Line(r)$ holds, then r cannot be (at least in the same segment) a road with a double centre line comprising one continuous and one broken line (*sign 296*); if z is a stationary vehicle, z cannot be a non-stationary vehicle etc. These inter-dependencies are usually represented by using so-called *ontologies* (Leone et al., 2022; Van Engers et al., 2008), which are basically first-order theories containing the rules of these inter-dependencies. In theory, such ontologies should be as detailed as possible, so as to be able to represent all the relevant factors included in legal norms. For example, following German courts and legal dogmatics, the minimal distance a driver must observe when passing by a stationary vehicle depends on several factors such as the width of the road, the type of the stationary vehicle, and whether there are visible people inside it or not. Ideally, one should employ an ontology containing all these concepts. Yet, using overly detailed formalisms might lead to implementation problems. In its core, verifying the legality of a manoeuvre with respect to a set of formalised norms is reducible to a *SAT problem* (or rather to a *satisfiability modulo theories* problem, cf. Lin & Althoff, 2022), i.e., checking whether a set of propositions is jointly satisfiable under a set of assumptions. However, SAT problems belong to the category of computationally expensive, so-called *NP-complete* problems. In general, adding more details to the formalism will lead to an exponential growth of the computational power required to verify the legality of a manoeuvre, which will quickly overwhelm even the fastest of modern computers. This is particularly problematic because, ideally, an autonomous vehicle should be able to consider various possible manoeuvres and trajectories in a relatively short amount of time, instead of analysing only one possibility *ad nauseam*. Moreover, an autonomous vehicle would not only have to be able to operate with all these concepts, but also to *detect* the existence of corresponding objects in its environment. In the example illustrated in Fig. 1, before it could even begin assessing the legality of following the trajectory m_1, AV would have to detect that it is driving on a road with a solid line and that there is an object $\mathfrak{O}$ obstructing its path. Then, it would have to determine what kind of object $\mathfrak{O}$ is. If it is a stationary vehicle, it would have to detect what kind of vehicle it is, and whether there are visible people inside it etc., which would require considerable computational power.

These examples show that formalising legal norms for the development of law-abiding machines is a complex interdisciplinary task. Legal norms are not just simply derived from written legislation. With the increasing degree of detail given in a formalisation, the computational resources for the calculation of the corresponding reasoning problems might also increase drastically. The formalisation process itself starts with legal scholars delivering a description of what the law actually is, i.e., of the legal norms proper, it continues with logicians representing these norms in a logical formalism, and ends with computer scientists and engineers implementing this formalism in a practically feasible manner. This interdisciplinary work-flow is illustrated in Fig. 2.

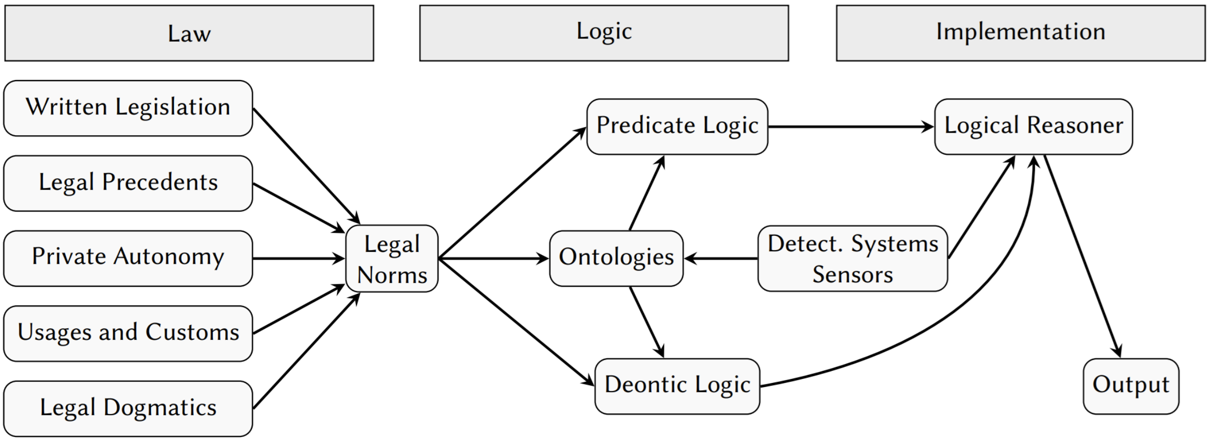


Fig. 2 – Representation of the work-flow from the analysis of the sources of law to the final implementation of legal norms in a law-abiding machine.

In this process, logicians see themselves caught between the need to adequately formalise the scientifically accurate norm descriptions given by legal scholars and the practical requirements and limitations determined by computer scientists and engineers. In particular, legal scholars tend to prefer a wider, *open-texture* formalisation, i.e., one that employs vague general concepts, thus being closer to a natural-language description of the respective norms. Computer scientists require variables and constants pertaining to a given domain to be assigned to these concepts, and engineers require these variables and constants to correspond to real-world objects which they can effectively detect. Hence in practice, more often than not, the formalisation first agreed upon between the involved parties has to undergo several modifications before it can actually be implemented. Furthermore, the use of many different formalisms and systems, e.g., formal languages, ontologies, logical reasoners, theorem provers, detection and driving systems along these different steps, with each possibly being based on different abstraction levels, often requires translating from one formalism into the other. This could eventually result in unnoticed errors, especially since the various employed formalisms might differ in their inherent expressiveness. Finally, having to constantly translate between formalisms also hinders the interdisciplinary communication between the many partners involved in developing law-abiding autonomous systems.

3. Related Work

Representing legal norms in a *normal form* constitutes a promising solution to the problems discussed above, as this could greatly simplify and unify the formalisation process with its various existing frameworks. As long as this normal form is at least partially based on elements of legal theory, it could also present a middle ground for legal professionals, logicians, computer scientists and engineers. In the following, Sec. 3.1 briefly discusses already existing widely applicable meta-formalisms for representing legal norms and their shortcomings with respect to interdisciplinarity. Sect. 3.2 then briefly discusses related work in the field of legal visualisation.

3.1. Meta-Formalisms for Legal Norms

Eclectic *meta-formalisms* for representing legal norms – i.e., symbolisms applicable to a wide range of underlying logical formalisms – already exist. For example, especially if norms are represented using (variations of) classic predicate logic, in particular first-order logic, one could employ the symbolism of the *Thousands of Problems for Theorem Provers (TPTP)* library (Sutcliffe, 2017). Beyond its overall flexibility, the TPTP-symbolism is both fairly easy for humans to read and compatible with `PROLOG`, which facilitates further implementations. Another good example is *LegalRuleML* (Palmirani et al., 2011), an XML-based annotation framework capable of directly representing legal norms and several properties usually associated with normativity, especially *defeasibility*. One could also, as proposed by Steen and Fuenmayor, 2022, attempt to combine both TPTP and LegalRuleML. However, while these meta-formalisms are extremely useful, they were evidently crafted with the implementation part of the process in mind. For the average legal scholar – and even for logicians and technicians not used to working with them –, they remain relatively complex and unintuitive. Hence, they do not facilitate the interdisciplinary communication between legal scholars, logicians, computer scientists and engineers. The normal form discussed here aims at filling this gap. It is far more primitive than the remarkable meta-formalisms mentioned above, and it focuses more on didactic aspects, on clarity, and on promoting and facilitating communication between humans. Hence, it is obviously not conceived as an alternative to implementation-oriented meta-formalisms such as TPTP's language or LegalRuleML, but as a human-focused methodological complement to them.

3.2. Legal Visualisation

Furthermore, the normal form proposed here was designed to be employed together with the legal visualisation method of *normative diagrams* (Sasdelli & Trivisonno, 2023), which is introduced in more details below in Sec. 4.2. Combining a textual formalism (like the normal form) with a visual one (like the diagrams) offers several advantages. Legal visualisations can be defined as human-oriented, (semi-) formal, pictorial (rather than verbal) knowledge management structures or *visual representations* in a general sense (Cyras & Lachmayer, 2023). Overall, depending on the goal pursued, visualisations found in the literature display different degrees of abstraction and formalism. Less formalised or structured imagery is usually employed for didactic (especially mnemonic) purposes (e.g., McCloskey, 1998), as well as for communication with children (Kessel, 2014), laymen (Carnein et al., 2014) and the public in general (e.g., in the context of the so-called *Proactive Law Movement* and *Legal Design*; cf. Murray, 2021); for an overview, cf. also Brunschwig, 2014. More formal methods, e.g., decision trees, graphs and the normative diagrams discussed here, are used not only within legal higher education, but also for theoretical and practical purposes (Kohlhase et al., 2021; Muff & Fill, 2021; Röhl & Ulbrich, 2007; Sasdelli & Trivisonno, 2023). As an intermediate abstraction level between natural language and a logical formalism (Lachmayer & Cyras, 2021), visualisations also constitute a valuable extension to legal tech applications, by facilitating both the interdisciplinary communication during the development of such applications as well as their later use by legal experts and the public in general. A good example is the `PROLOG`-based practical legal reasoning system `PROLEG` (Satoh, 2023), which in more user-friendly implementations also generates a decision tree diagram illustrating the reasoning process leading to the decision derived by the program. A new trend in the field involves the usage of virtual reality tools (Muff & Fill, 2021; Muff et al., 2022).

4. Normal Form and Normative Diagrams

The following Sec. 4.1 gives an in-depth description of the proposed normal form and also considers its advantages in light of different existing legal formalisms. Sec. 4.2 shows how *normative diagrams* can be used to visualise the normal form and thus further facilitate the comprehension for the involved parties of the formalisation process.

4.1. Normal Form Structure and Theoretical Background

In general terms, one can distinguish between two main types of legal norms (Kelsen, 1979, pp. 76-78):

1. Norms that refer to the validity of other norms, e.g., procedural law, norms regulating the legislative process and derogative norms;
2. Norms that refer directly to the actions or to the overall behaviour of agents.

In the following, only norms of the latter type shall be considered. Such norms can be further subdivided into three categories: (1) prohibitions, (2) obligations, and (3) permissions (Kelsen, 1979, pp. 76-77, Bobbio, 1993, pp. 96-99). Each of these categories is connected to different violation conditions. Prohibitions, for example, are violated when a case corresponds to the norm's content or, in case of analogy, when a case is sufficiently similar to the norm's content. Determining the violation conditions for obligations is a more difficult task. Generally speaking, it seems reasonable to assume that obligations are violated when it becomes impossible to satisfy the norm's content. This would indicate an important temporal element to the definition of the violation conditions of obligations, which, however, would become a challenge when considering obligations that can never be completely fulfilled (even without being thus necessarily violated). Examples would be very general legal or moral principles, in particular so-called *imperfect duties* (Johnson & Cureton, 2024), like the duty to respect human dignity, the duty to live a healthy life etc. Further challenges concerning the definition of the violation conditions of obligations could arise from the concept of supererogation, i.e., roughly speaking, doing more than one's duty actually requires (Heyd, 2023; Wessels, 2002). For simplicity, in the paper at hand, it shall be assumed, as it is done in some works of legal theory (cf., e.g., Kelsen, 1979), that obligations can be reformulated as prohibitions. Finally, Permissions cannot be violated.

Intuitively, norms regulate sets of actions or behaviours that:

- belong to specific action or behaviour categories;
- happen at a given place;
- are performed by specific agents;
- transpire at a specific point in time.

Hence, irrespective of its category, the content or *validity domain* of a norm can be seen as stretching throughout four *dimensions*: object (O), space (R), subject (S) and time (T) (cf. Bobbio, 1993, p 214, Kelsen, 1979, pp. 116-119, Nawiasky, 1948, p. 85). When considering only prohibitions, the most obvious violation condition of a norm coincides with a description of its validity domain in the four dimensions mentioned above: a case will constitute a violation to the norm if it falls (or is subsumed) under its normative content or validity domain, i.e., if it is subsumed under the category of actions which the norm directly state as being prohibited. Further violation conditions could still be derived, e.g., by considering analogy arguments. Setting analogies aside and focussing on subsumption, the conditions for the violation V_ϕ of the norm ϕ can be determined by:

$$V_\phi \Leftarrow (O_\phi \wedge R_\phi \wedge S_\phi \wedge T_\phi) \tag{3}$$

Where $\Leftarrow$ is a *necessary condition* operator and O_ϕ, R_ϕ, S_ϕ and T_ϕ are high-level descriptions respectively of the objective, spacial, subjective and temporal dimensions of the norm ϕ's validity domain.

The structure of the equation (3) can be used as a normal form for logic-based representations of legal norms. This claim is based on the assumption that every norm can be adequately represented by its violation conditions. This is fairly obvious, under the assumptions made above, for prohibitions and obligations. For *permissions*, which, as stated above, cannot be violated, it seems reasonable to establish the convention that their representation in this normal form corresponds to an empty expression. This means that permissions will be basically ignored, except when dealing with them as exceptions to other norms. This should not constitute a problem for the applications discussed here: when developing a law-abiding autonomous agent, it is more convenient to only keep track of the actions which the machine should avoid, i.e., of which actions constitute violations – all other actions (which are evidently way larger in number) can then be considered to be permitted. Moreover, permissions constituting exceptions to prohibitions or obligations can be considered as (partial) *derogations*, i.e., as limitations of a norm's violation conditions.

The normal form promotes a natural formalisation process. One first starts with the high-level descriptions O_ϕ, R_ϕ, S_ϕ and T_ϕ of the violation conditions of the norm ϕ, which basically just state that the conditions for a case to fall under the norm's validity domain and thus be violated (if the norm is a prohibition norm) are satisfied. Then, one can, by means of interdisciplinary work between legal scholars, computer scientists and engineers, introduce further, more detailed descriptions of the circumstances that satisfy each of these high-level descriptions. The possibility to reduce such different formalisms back to one and the same normal form demonstrates the flexibility of the presented method. Since each high-level description is directly associated with one of the norm's validity dimensions, using the normal form leads to a better organisation of the formalisation work. Ideally, one would then be able to more easily identify which elements of the legal norm (as described by legal scholars) cannot (yet) be efficiently processed or detected by the available methods employed (as determined by computer scientists and engineers). This could facilitate the search for approximations or other alternative solutions.

Finally, besides being based on the legal-theoretical division of a norm's validity domain in four dimensions, the normal form presented here was also conceived to be easily combined with existing formalisms like the ones just mentioned above. For example, the necessary condition operator $\Leftarrow$ was chosen because of the long tradition of using *horn-clauses* to formalise legal norms in `PROLOG` or related formalisms (cf., e.g., Satoh, 2023; Sergot et al., 1986). Due to its relatively simple structure, it can be easily combined with any formalism based on predicate logic, especially with temporal logics (Maierhofer et al., 2022). The normal form can also be combined with formalisms based on modal logic, especially with deontic logic (Gabbay et al., 2013). This can be done, e.g., by conditioning the fulfilment of one or more predicates on the satisfaction of formulae under given conditions represented in the desired formalism. Overall, the normal form offers an elegant way of both organising the designed formulae and guiding the further development of the formalisation process, while also enabling a combination of various different formalisms. This can be employed to determine adequate conditions for the satisfaction of the respective predicates culminating in the normal form's four conjuncts.

4.2. Enhancing the Normal Form with Normative Diagrams

Like the normal form presented here, normative diagrams are also based on the idea that a norm's validity domain extends over four dimensions. By assigning each dimension to an axis in a coordinate system, the validity domain of a norm can be represented as a region in a four-dimensional metric space. For example, if it is assumed that the action of crossing or straddling a solid line is delimited by the values o_1 and o_4; the set of all German roads with solid lines by the values r_1 and r_3; the set of all drivers by the values s_1 and s_2; and the set of all possible points in time by $\{t_x|x \in \mathbb{N}\}$, then the validity domain U_ψ of the norm associated with sign 295 of the German StVO (let this norm be called ψ) can be defined as being the set:

$$U_\psi = \{ < o_a, r_b, s_c, t_d > \mid o_a \in [o_1, o_4] \wedge r_b \in [r_1, r_3] \wedge s_c \in [s_1, s_2] \wedge t_d \in \{t_x|x \in \mathbb{N}\} \} \tag{4}$$

Since the norm is valid at all points in time, i.e., it is *always* prohibited to cross or straddle the line, the temporal dimension can be omitted. Hence, it is possible to represent the norm ψ in a three dimensional normative diagram, as illustrated in Fig. 3. The region delimited by the red three-dimensional figure corresponds to ψ's validity domain. Here, the colour red is used to indicate that ψ is a prohibition norm. Similarly, the colours blue and yellow can be used to represent obligation and permission respectively. Concrete cases can be depicted as points in this three-dimensional metric space. Since the norm ψ is a prohibition, if a point is located inside of the three-dimensional cuboid of the norm, the corresponding case falls under its validity domain and thus *violates* the norm. Hence, the violation conditions specified by the *normal form* can be easily visualised through respective normative diagrams, while the concrete values on the axes can help in defining the normal form for a norm.

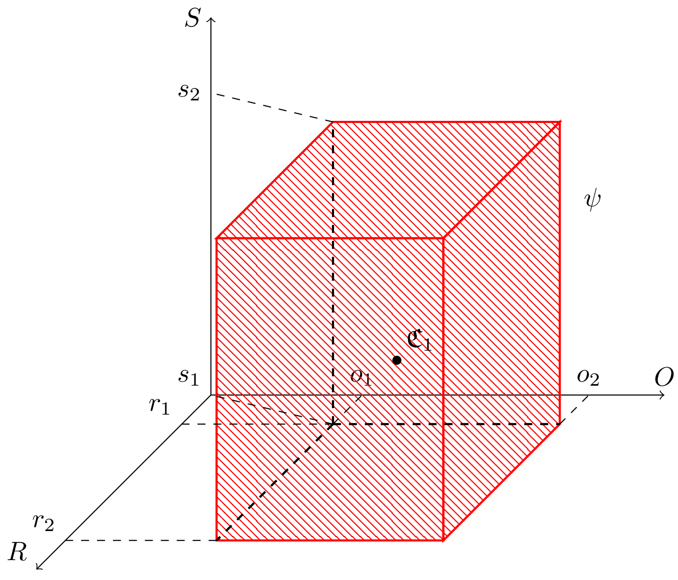


Fig. 3 – A three-dimensional normative diagram for the norm ψ with case $\mathfrak{C}_1$ violating the norm.

For simplicity, one could also restrict the analysis of the subjective dimension of the norm's validity domain to the set of all *drivers*. In this case, it would be possible to also omit the subjective dimension of the norm's validity domain; for it is in fact prohibited for *all drivers* to cross or straddle the solid line. In this case, a two-dimensional representation would suffice (left side of Fig. 4). Finally, one could also restrict the analysis to the set of roads with solid lines, because, evidently, one can only cross or straddle these lines while driving on roads that have them. Hence, the norm could also be adequately represented with a one-dimensional diagram containing only the objective dimension (right side of Fig. 4).

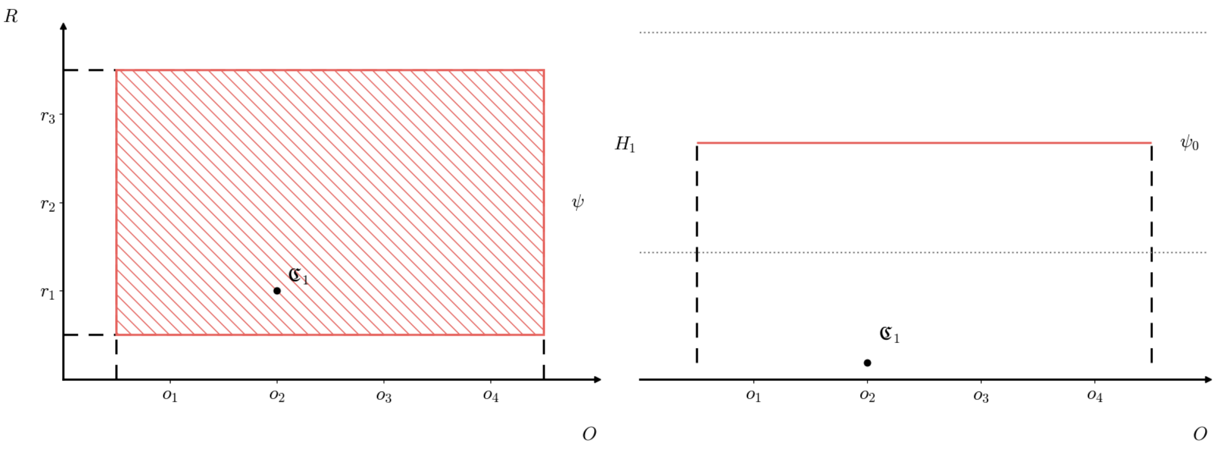


Fig. 4 – Simplified diagrams for ψ with two dimensions (left) and with one dimension (right), each with case $\mathfrak{C}_1$ violating the norm.

The *unwritten* exception to the solid line norm which allows drivers to cross the line when passing by a road narrowing, an obstacle or a stationary vehicle in the sense of § 6 of the StVO can be represented by adding a second norm ζ to the diagram, as shown in Fig. 5. Since the validity domains of the norms intersect, i.e., since $U_\psi \cap U_\zeta \neq \emptyset$, and since the norms differ in their mode, i.e., ψ is a prohibition and ζ is a permission, these norms constitute a *normative conflict* or a (legal) *antinomy* (Bobbio, 1993, pp. 209-216, Engisch, 1985, p. 160). Moreover, since $U_\zeta \subset U_\psi$, this conflict is of the type *total-partial*, and can thus be solved by the classic speciality criterion (*lex specialis derogat legi generali*). When representing the antinomy between ψ and ζ, it is useful to employ a two-dimensional diagram (left side of Fig. 5), because one could argue that the exception contained in ζ does not apply to all roads with solid lines, but only to those which are wide enough to allow for the vehicle to pass through.

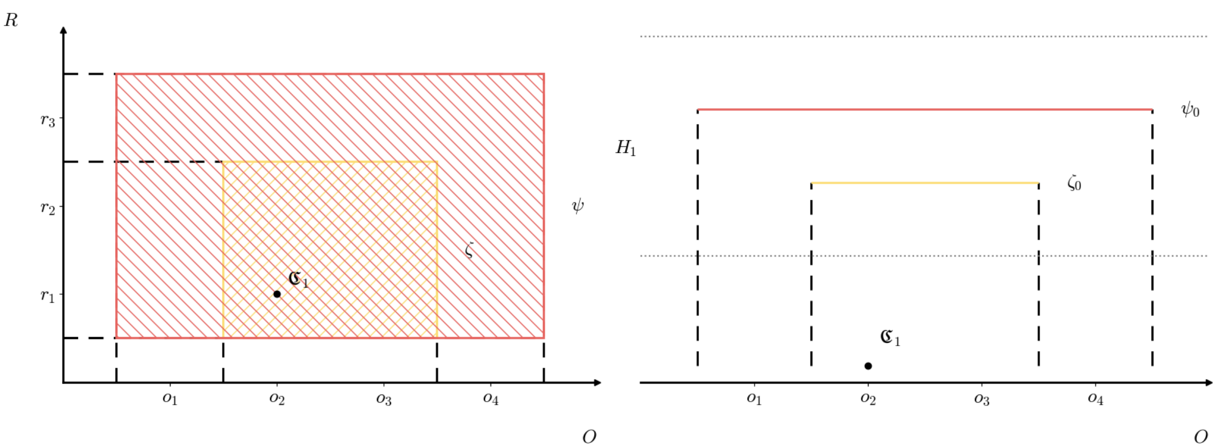


Fig. 5 – The unwritten exception to the solid line norm can be represented as a legal antinomy between the norms ψ and ζ. The values between o_2 and o_3 correspond to the action of crossing a solid line while passing by an obstacle, a road narrowing or a stationary vehicle in the sense of § 6 of the StVO. The values between r_1 and r_2 represent those roads that are wide enough to allow for the manoeuvre to be performed. Evidently, this will depend on the object obstructing the way and on the width of the passing vehicle.

Normative diagrams can also be employed to represent legal arguments, e.g., analogy, appeals to the contrary, as well as more complex aspects of legal norms, e.g., their hierarchy level in the legal order (for details, cf. Sasdelli and Trivisonno, 2023). Alternative representation forms that allow the complete visualisation of all four dimensions of the validity domain in one single figure have also been developed (Steffes & Sasdelli, 2025). Being a simple but nonetheless efficient way of visualising legal norms and other legal phenomena, these diagrams could constitute a valuable component in a user-friendly *legal norm formalisation tool*. This tool could be employed in a wide range of different applications, e.g., for research and development purposes in the area of law-abiding autonomous agents or for didactic purposes in law faculties or in the field of legistics. Such tool could be developed on the basis of `ViNo`, an open-source tool[1] that automatically generates normative diagrams from given inputs (a prototype was presented by Steffes and Sasdelli, 2024).

5. Example

This section now presents a precise example on how to apply the presented normal form with a particular focus on its combination with normative diagrams. For this example, the manoeuvre m_1 depicted in Fig. 1 shall be considered again.

The legality of m_1 depends on a set of norms $\mathfrak{N}$ containing the norms applicable to m_1. In this case, it consists of the norms corresponding to sign 295 and to § 6 of the German Road Traffic Regulation, as well as further prescriptions such as speed limits or norms related to the general obligation to drive carefully and to be mindful of other road users (§ 1 of the German Road Traffic Regulation). For simplicity, assume $\mathfrak{N} = \{\psi,\ \chi,\ \kappa\}$, i.e., $\mathfrak{N}$ contains only the three norms ψ, χ and κ. These norms correspond respectively to the prohibition of crossing the solid line (sign 295, ψ), to the regulation of the manoeuvre-type *passing* (§ 6, χ), and to the speed limit of road r, which can be assumed to be 30 km/h (κ). In the normal form, each of these norms can be represented through the expression

$$V_x \Leftarrow (O_x \wedge R_x \wedge S_x \wedge T_x). \tag{5}$$

Assuming $[m_1]$ to be a comprehensive description of the manoeuvre m_1 in some logic formalism, checking whether the manoeuvre m_1 is legal can be reduced to the task of checking whether the expression

$$(O_x \wedge R_x \wedge S_x \wedge T_x) \Leftarrow [m_1] \tag{6}$$

is satisfied for any $x \in \mathfrak{N}$. Following the structure of the normal form, the expression $[m_1]$ can be further divided in the subformulae O_{m_1}, R_{m_1}, S_{m_1} and T_{m_1} which each contain the predicates for detecting a violation in one of the four dimensions:

$$(O_x \wedge R_x \wedge S_x \wedge T_x) \Leftarrow (O_{m_1} \wedge R_{m_1} \wedge S_{m_1} \wedge T_{m_1}) \tag{7}$$

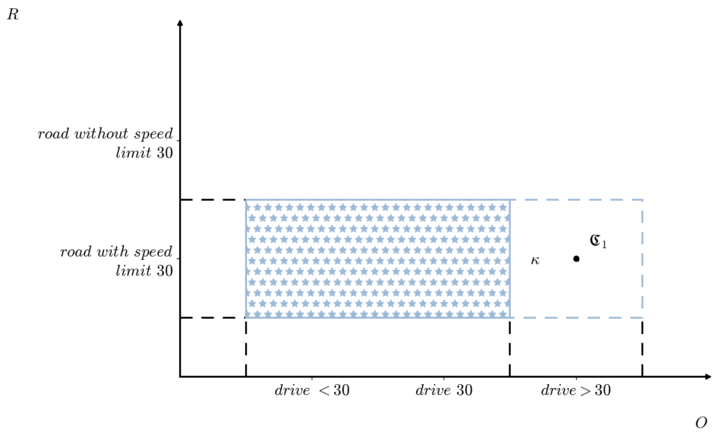
For this, it is first necessary to further define both $[m_1]$ and the norms in $\mathfrak{N}$. In particular, a more detailed definition of the norms in $\mathfrak{N}$ – to be provided by legal scholars – should clarify which kinds of information $[m_1]$ should ideally contain – i.e., as provided by the sensors of an AV – to determine the violations.

The speed limit κ can be assumed to be valid at all times for all subjects with respect to all manoeuvres they perform. Thus, the norm is violated by all manoeuvres performed while driving faster than 30 km/h on road r (this ignores, for the sake of simplicity, the obvious exception given to emergency vehicles, e.g., ambulances). Hence, one can further specify the violation condition of κ by determining that $T_\kappa \Leftarrow \top$ and $S_\kappa \Leftarrow \top$ hold, where $\top$ is any logical tautology. For the further conditions, one can determine, e.g., $R_\kappa \Leftarrow Road_{30}(m)$ and $O_\kappa \Leftarrow f_{speed}(m) > 30$, where m is a variable ranging over all possible manoeuvres, $Road_{30}$ is a predicate that is satisfied if and only if a manoeuvre is performed on a road with a speed limit of 30 km/h, and f_{speed} is a function that retrieves the maximum speed of a given manoeuvre.[2] These thoughts are illustrated with example values in Fig. 6 when considering κ to be an obligation, i.e., the obligation of driving in a speed below the set limit. Following the assumption made above that an obligation may also be generally reformulated as a prohibition, Fig. 7 depicts the norm as a prohibition, i.e., the prohibition of driving in a speed above the set limit.

The solid line norm ψ can be likewise assumed to be valid at all times and with respect to all subjects. As discussed above, its spatial violation condition only refers to roads containing solid lines; hence, one can further determine $R_\psi \Leftarrow R\_Sld\_Line(m)$. At first glance, its objective dimension refers to all possible manoeuvres (e.g., overtaking, turning left, making a U-turn) that imply crossing or straddling the solid line. However, as discussed above, the norm corresponding to § 6 (*Passing*) of the German Road Traffic Regulation contains

[1] The code can be found at https://github.com/bs-000/norm-visuals. The repo also contains a Windows executable file to provide an easy access for technical laymen.

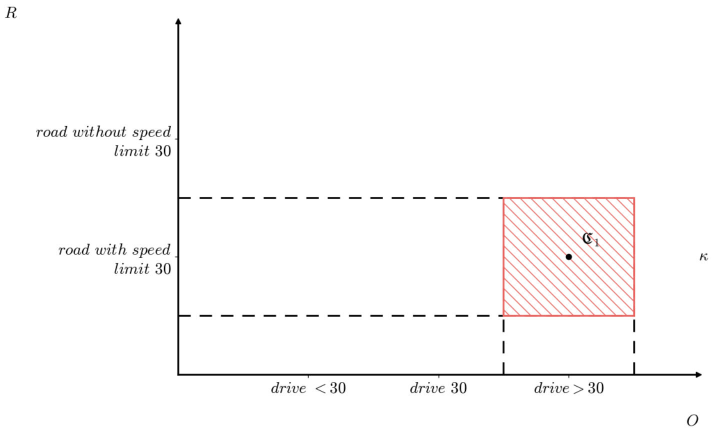
[2] Alternatively, one could replace the predicate $Road_{30}$ by a respective function f_{road} that retrieves the road (or rather, a road-index $[r] \in \mathbb{N}$ functioning as a name for the road) in which a given manoeuvre is performed. In this case, one would write the respective condition as $R_\kappa \Leftarrow f_{road}(m) = [r]$.



$$V_\kappa \Leftarrow (O_\kappa \wedge R_\kappa \wedge S_\kappa \wedge T_\kappa)$$

$$O_\kappa \Leftarrow f_{speed}(m) > 30$$
$$R_\kappa \Leftarrow Road_{30}(m)$$
$$(R_\kappa \Leftarrow f_{road}(m) = [r])$$
$$S_\kappa \Leftarrow \top$$
$$T_\kappa \Leftarrow \top$$

Fig. 6 – Diagram and normal form description of the speed limit of 30 km/h with κ as an obligation.



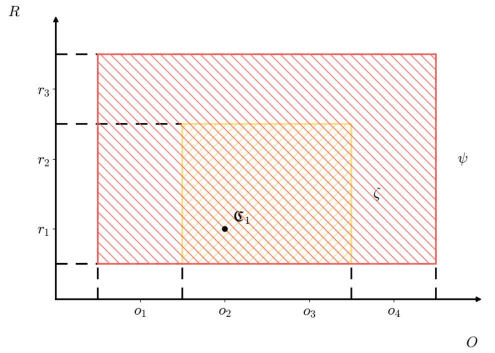
$$V_\kappa \Leftarrow (O_\kappa \wedge R_\kappa \wedge S_\kappa \wedge T_\kappa)$$

$$O_\kappa \Leftarrow f_{speed}(m) > 30$$
$$R_\kappa \Leftarrow Road_{30}(m)$$
$$(R_\kappa \Leftarrow f_{road}(m) = [r])$$
$$S_\kappa \Leftarrow \top$$
$$T_\kappa \Leftarrow \top$$

Fig. 7 – Diagram and normal form description of the speed limit of 30 km/h with κ as a prohibition.

an implicit exception to this rule: when performing the manoeuvre *Passing*, drivers are allowed to cross the solid line. Hence, one could determine $O_\psi \Leftarrow m \in \mathfrak{M}_{xsl} \backslash \{Passing\}$, where $\mathfrak{M}_{xsl} \backslash \{Passing\}$ is the set of all manoeuvres that imply crossing or straddling the solid line except the manoeuvre *Passing*. To visualise this rule-exception, one can interpret § 6 StVO to also contain (or at least imply) a permissive norm ζ, stating that it is allowed to cross or straddle the solid line when passing by. The resulting normative conflict between ψ and ζ would be solvable by the collision rule *lex specialis*, which, in effect, leads to the same result as stating that *passing* manoeuvres constitute exceptions to the general solid line rule. These thoughts are illustrated in Fig. 8.

Finally, a direct interpretation of § 6 StVO also entails an obligation norm χ, which regulates the *Passing* manoeuvre. This norm can be specified as follows. As it was the case with κ and ψ, χ's temporal and subjective violation conditions can be assumed to be always fulfilled. Its spatial condition concerns roads wide enough so that *Passing* is possible. Hence, one could determine: $R_\chi \Leftarrow Rd\_Width\_Ok(m)$. § 6 StVO stipulates that drivers passing by an obstacle have to allow for oncoming vehicles to pass through. Hence, by means of a still fairly high-level description, the objective violation condition could be specified as $O_\chi \Leftarrow Impedes\_Onc\_Traffic(m)$. Evidently, the predicates Rd_Width_Ok and $Impedes\_Onc\_Traffic$ are still too abstract to allow for meaningful practical implementations. Therefore, it would be necessary to further specify the conditions under which these predicates are satisfied. For example, one could further define $Road\_Width\_Ok$ as a function of the overall width of the road, the width of the vehicle performing the respective manoeuvre and the width of the obstacle in question. However, even after the addition of these specifications, this representation of the norm regulating the *Passing* manoeuvre would still be missing an important point: while this is not explicitly stated in the text of § 6 StVO, a deeper analysis of legal precedents and legal dogmatics reveals that *Passing* is only allowed when the manoeuvre is performed while observing a *minimal safety distance*, which depends on the circumstances of the concrete case. Hence, it would be more appropriate to represent χ's objective violation condition as $O_\chi \Leftarrow (Impedes\_Onc\_Traffic(m) \vee$

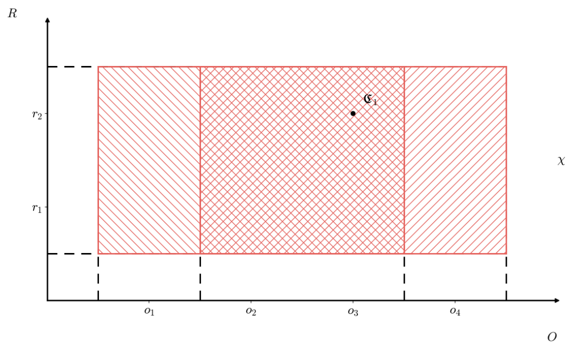


$$V_\psi \Leftarrow (O_\psi \wedge R_\psi \wedge S_\psi \wedge T_\psi)$$

$$\begin{aligned} O_\psi &\Leftarrow m \in \mathfrak{M}_{xsl} \backslash \{Passing\} \\ R_\psi &\Leftarrow R\_Sld\_Line(m) \\ S_\psi &\Leftarrow \top \\ T_\psi &\Leftarrow \top \end{aligned}$$

Fig. 8 – Diagram and normal form description of the solid line rule ψ. The implicit exception derived from the *passing* rule is represented by ζ. Notice that, although they are both derived mainly from the same legal source (§ 6 of the German Traffic Road Regulation) ζ is different from χ.

$Violates\_Safety\_Distance(m)$). This is illustrated in Fig. 9.



$$V_\chi \Leftarrow (R_\chi \wedge T_\chi \wedge S_\chi \wedge O_\chi)$$

$$\begin{aligned} O_\chi &\Leftarrow Impedes\_Onc\_Traffic(m) \vee \\ &\quad\ Violates\_Safety\_Distance(m) \\ R_\chi &\Leftarrow Rd\_Width\_Ok(m) \\ S_\chi &\Leftarrow \top \\ T_\chi &\Leftarrow \top \end{aligned}$$

Fig. 9 – Diagram and normal form description of the norm χ regulating the *Passing* manoeuvre (§ 6 of the German Road Traffic Regulation). The area between $o_{1'}$ and $o_{1''}$ corresponds to manoeuvres that satisfy both predicates $Impedes\_Onc\_Traffic$ and $Violates\_Safety\_Distance$.

Likewise, it would be necessary to further specify the predicate $Violates\_Safety\_Distance$. According to legal scholarship, the *minimal safety distance* required when performing the manoeuvre *Passing* usually amounts to one meter (König, 2023, Rn. 7). However, depending on the circumstances, the required distance can also be wider or smaller. Among the most commonly considered examples are those in which the obstacle is another vehicle. *Prima facie*, a driver (or an AV) should detect the type of said vehicle, e.g., whether it is a truck, a bus or a car, whether one of its doors is open, or if there are people in the vehicle or around it. German judicial practice establishes more specific provisions concerning the minimal safety distance in such situations. For instance, in case a person is likely to exit or to open the door of a vehicle, drivers are usually expected to keep a distance of more than one meter when passing (Gutt, 2022, Rn. 14, König, 2023, Rn. 7)(cf. also Schauseil, 2011). Similarly, when passing by a bus, a garbage truck or construction machines, drivers have to take into account that people might suddenly walk onto the street. Hence, they are required to drive slow enough to be able to quickly stop if necessary, or to generally keep a larger safety distance, e.g., of 1,8-2 m (Heß, 2022a, Rn. 7-8, Bender, 2016, Rn. 8) (cf. also the decision BGHSt 13, 169). On the other hand, a distance of 50 cm may be enough if the road is particularly narrow (König, 2023, Rn. 7). These concrete standards could be implemented by individually assigning the respective safety distance to each manoeuvre, so that $Violates\_Safety\_Distance(m)$ is satisfied whenever the actual distance is smaller than the allowed distance. However, it is important to notice that these criteria are not absolute; they serve only as guidelines to help determine the minimal safety distance in a concrete situation.

This approach reveals a further problem: a review of the sources related to § 6 of the German Road Traffic Regulation can only reveal concrete requirements for a limited number of situations, i.e., the situations already considered by German courts or by legal dogmatics. Yet, specifying χ requires the definition of a safety distance for *all* possible situations. Thus, to avoid violating χ, the specific examples need to be generalized so that safety distances are assigned to situations that have not been explicitly included. For example, case law and literature have not specifically addressed the distance required when passing a stationary delivery truck, which can be seen as analogous to passing a stationary garbage truck – since one could likewise expect that people might suddenly walk onto the street – and would therefore require a similar distance.

One approach to solving this issue is to gather characteristics of known cases and have the AV compare unknown cases to these standards. In the example above, the safety distance would then be the same as when passing a stationary garbage truck in all cases in which the sensory data for the obstacle and the surrounding pedestrians is sufficiently similar to the data associated with garbage trucks. Thus, an unlimited number of situations could be accounted for even with a limited number of sources. The assessment of a similarity predicate could be done by means of a logical formalism designed specifically for representing degrees of similarity and analogy. An example could be so-called *neighbourhood frames* (cf., e.g., Nortmann, 1989). A different approach could be to employ the visual form of the normative diagrams. If the values on each of the dimensions' scales can be comprised to a cardinal scale, distance measures like the euclidean distance or the Hamming distance may be employed to measure the similarity of different cases or the distance between a norm and a case. A smaller distance may be interpreted as a higher similarity. Of course, determining the optimal degree of similarity (in terms of both engineering and legal standards) and creating these orderings are challenging tasks that lie beyond the scope of the paper at hand.

6. Conclusion and Future Work

The normal form presented here is a valuable methodological tool for the complex and interdisciplinary task of formalising legal norms. Its structure allows for a multi-step and multi-level formalisation procedure. Starting with very high-level descriptions of the violation conditions of a norm – organised in a way that is based on legal theory, i.e., on the four dimensions of a norm's validity domain –, it facilitates that gradual further specification to more detailed descriptions. This framework is particularly flexible: with each further specification, a new formalism (e.g., different types of modal, temporal or deontic logic) may be employed according to the respective needs. Such a structure allows for a better understanding of the representation and implementation challenges one might face when formalising legal norms. Finally, especially when combined with normative diagrams, the normal form facilitates the visualisation of the legal norms to be formalised, as well as of more complex legal-theoretical phenomena.

An important future work topic concerns dealing with the formalisation of legal gaps, legal antinomies and of contrary-to-duty-situations within the framework of the normal form. In particular, situations in which at least one norm has to be violated, i.e., situations in which all possible manoeuvres in a set $\mathfrak{M}$ lead to some violation V_x of a norm $x \in \mathfrak{N}$, could be solved by establishing a priority degree between the violations. This priority degree could be based on elements of legal theory, e.g., the classic collision rules for solving legal antinomies (Bobbio, 1993, p. 218-222). A further topic for future work on theoretical aspects is the definition of specific criteria for determining the distance between different case-descriptions as well as between a case-description and a norm-description, which would serve as a basis for the formalisation of analogy arguments. Finally, more research on the particularities concerning the violation conditions of obligations (e.g., aspects concerning imperfect duties and supererogation) is also needed. Additionally, the visualisation of four-dimensional objects may be challenging to understand for humans as discussed by Steffes and Sasdelli, 2025. From a more technical perspective, the main topic for future work involves developing an interactive legal formalisation tool implementing the normal form framework presented, while also combining it with existing logical formalisms for legal knowledge representation and with methods of legal visualisation, e.g., with normative diagrams.

Data/Software Access Statement

For the generation of the images displayed in this paper the open source tool ViNo (https://github.com/bs-000/norm-visuals) was utilized.

Contributor Statement

Author 1: Conceptualisation, Formal Analysis, Methodology, Visualisation, Writing – Original Draft, Writing – Review & Editing. Author 2: Formal Analysis, Software, Visualisation, Writing – Original Draft, Writing –

Review & Editing. Authors 3-5: Investigation, Writing – Original Draft.

Use of AI

During the preparation of this work, the authors used no AI-Tools. The authors take full responsibility for the content of the publication.

Conflict Of Interest (COI)

There is no conflict of interest.

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